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PASSIVITY BASED CONTROL OF IRREVERSIBLE PORT HAMILTONIAN SYSTEM: AN ENERGY SHAPING PLUS DAMPING INJECTION APPROACH

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TRABAJO DE TESIS, presentado en cumplimiento parcial de los requisitos para el título de Ingeniero Civil Electrónico y el grado de Magíster en Ciencias de la Ingeniería Electrónica de la Universidad Técnica Federico Santa María.

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dedicado a mi tía

a mi brujita

a mi pichonga

y a mi lelita

las amo por siempre.

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CONTENTS

RESUMEN

Los sistemas puerto Hamiltonianos irreversibles (IPHS) son una extensión de la clásica formulación de sistemas puerto Hamiltonianos (PHS). Al igual que los sistemas PHS, esta forma de modelado permite representar una gran cantidad de sistemas multifísicos, con la capacidad de representar cada sistema físico como un bloque capaz de conectarse con los demás a través de funciones de energía. A diferencia de un sistema PHS, los IPHS representan en su estructura el primer principio de la termodinámica (conservación de la energía) y el segundo principio termodinámico (la creación irreversible de entropía). Esta representación pues, permite no solo el modelado de sistemas electromecánicos sino que también permite representar a sistemás termodinámicos y en general, sistemas con procesos irreversibles. Este formalismo, al igual que en sistemas PHS, provee un marco teórico para el control de sistemas multifísicos.

Técnicas de control pasivas y no lineales han demostrado ser útiles en el control de sistemas PHS. Estas técnicas tienen como objetivo modificar la función de energía del sistema de forma tal que la función de energía resultante sea un candidato a función de Lyapunov, y tenga un mínimo estricto de energía en un punto de equilibrio deseado. Esta forma de control garantiza la estabilización del sistema en un punto de equilibrio deseado, junto con la estabilidad asintótica del sistema.

Dentro de estas técnicas de control pasivas, control por interconección y modelado de energía han sido utilizadas para cambiar el punto de equilibrio de una función de energía candidata a función de Lyapunov en sistemas PHS; la existencia y utilización de las funciones de Casimir resultan, por lo tanto, fundamentales para tal propósito pues estas funciones son invariantes estructurales del sistema. Si bien la estabilidad del sistema es garantizada mediante control por interconexión, la incorporación adicional de amortiguación, a través de la entrada pasiva del sistema, asegura que el sistema sea asintóticamente estable en el punto deseado.

El principal objetivo de esta tesis es extender las técnicas de control pasivas y no lineales utilizadas en sistemas PHS al control de sistemas IPHS. En particular, se propone un método sistemático de diseño de controladores para IPHS, basado en las técnicas de control por interconexión y la inyección de amortiguamiento. Para ello, se plantea una estructura de controlador IPHS como interconexión con el sistema, y se derivan condiciones para la existencia de invariantes estructurales que permitan mover el punto de equilibrio. En el proceso de diseño, resulta de gran importancia para el diseño de la función de energía, el concepto de función de disponibilidad. Esta función resulta ser un candidato a función de Lyapunov para un punto de equilibrio deseado, y estrictamente convexa.

El resultado es un método sistemático de diseño, haciendo uso de las técnicas clásicas de control pasivo como control por interconexión y energy shaping, junto con invariantes estruc-

turales de Casimir, y funciones de disponibilidad de energía; para sintetizar un controlador que estabiliza el sistema IPHS en un equilibrio dinámico deseado, y que es asintóticamente estable. Finalmente, se realizan simulaciones utilizando sistemas con procesos irreversibles-reversibles.

ABSTRACT

Irreversible Hamiltonian Port Systems (IPHS) are an extension of the classic Port Hamiltonian System (PHS) Formulation. Like PHS systems, this form of modeling allows the representation of a large number of multiphysical systems, with the ability to represent each physical system as a block capable of connecting with the others through energy functions. Unlike a PHS system, IPHS represent in their structure not only the first principle of thermodynamics (energy conservation) but also the second thermodynamic principle (the irreversible creation of entropy). Therefore, this representation allows not only the modeling of electromechanical systems but also allows to represent thermodynamic systems and, in general, systems with irreversible processes. This formalism provides. as in the case of PHS, a theoretical framework for the control of multiphysical systems.

Passive and non-linear control techniques have proven to be useful in controlling PHS systems. These techniques aim to modify the energy function of the system such that the resulting energy function is a candidate for a Lyapunov function, and has a strict minimum energy at a desired equilibrium point. This form of control ensures stabilization of the system at a desired equilibrium point, along with asymptotic stability of the system.

Within these passive control techniques, control by interconnection and energy shaping have been used to change the natural equilibrium point of a Lyapunov candidate energy function in PHS; the existence and use of the Casimir functions are, therefore, fundamental for this purpose since these functions are structural invariants of the system. Although the stability of the system is guaranteed by the energy-Casimir control, the additional incorporation of damping, through the passive input of the system, ensures that the system is asymptotically stable at the desired point.

The main objective of this thesis is to extend the passive and non-linear control techniques used for PHS to the control of IPHS. Precisely, a systematic design method control for IPHS is proposed, based on control by interconnection techniques and damping injection. For this, an IPHS controller structure is proposed as an interconnection with the system, and conditions are derived for the existence of structural invariants that allow changing the equilibrium point. In the design process, the concept of availability function is of great importance for the design of the energy function. This function turns out to be a candidate for a Lyapunov function for irreversible systems.

The result is a systematic design method, using classical passive control techniques such as control by interconnection and energy shaping, along with Casimir structural invariants, and energy availability functions to synthesize a controller that stabilizes the IPHS system in a specified dynamic equilibrium, and that is asymptotically stable. Finally, simulations are performed using systems with irreversible-reversible processes.

Chapter 1

INTRODUCTION

In this introductory chapter, the port Hamiltonian framework is recalled; we give the state of the art in the passivity based control techniques for the control of PHS. Subsequently, we mention extensions for the PHS framework to cope with systems that come from irreversible thermodynamics, as PHS are used for modelling and control of electromechanical systems in its classic definition. One of these extension is called irreversible port Hamiltonian systems (IPHS) which can be used to model multiphysical systems and therefore its definition can be exploited for control purposes.

The organization of the chapters and the main contributions of this thesis are also given.

1.1 Motivation and state of the art

The Port Hamiltonian system formulation (PHS) has been used for control and modelling of electrical, mechanical, and in general multiphysics systems ([22], [10], [36]) which are described by the first law of thermodynamic. The PHS framework formalizes the basic interconnection laws, e.g Kirchhoff laws in electrical systems or Newton laws in mechanical systems, together with the power preserving elements by a geometric structure using the energy between the elements as the interconnection, and defines the Hamiltonian as the total energy stored in the system.

Energy then becomes essential in modeling systems as PHS, since it relates variables that come from different physical domains. Since the PHS framework is defined by a Hamiltonian energy function of its power preserving elements, it has been largely used for the control of multiphysical domain systems ([36], [10]).

A key property for the control of PHS is the existence of the Casimir functions ([37], [10], [38]) which are structural invariants of the system.

Control by interconnection (Cbi) techniques aim at closing the PHS system in a loop with negative feedback, and with a controller that also has a PHS structure. In this way, the total energy of the system is represented by the sum of the energies of the controller and the PHS system. The controller energy is designed in such a way that the total energy of the system is a candidate for Lyapunov function and has a global minimum at some desired equilibrium point ([25],[27],[37]). By Casimir generation, the coordinates of the system are related to the Casimir function and the useful Casimir functions for the closed-loop system satisfy a set of partial differential equations (PDE) [37].

A proper choice of an energy controller function such that the Hamiltonian for the closed-loop system is a candidate Lyapunov function, guarantees stability of the system at the equilibrium point [37], while the asymptotic stability of the system is achieved by the **injection of damping (Di)**. A generalization of the energy-Casimir method is the

interconnection and damping assignment passivity based control (IDA-PBC) which has been developed in [26]. In the IDA-PBC control method, the closed-loop energy function is obtained directly through the resolution of a set of partial differential equations, through the choice of a desired controller and damping. Unlike control by interconnection, in IDA-PBC the energy is obtained directly from the resolution of a PDE, and not by completing squares or some other tentative method.

Port Hamiltonian system theory, as mentioned at the beginning of the section, has been successfully used for modeling electromechanical systems; that is, systems that can be modeled by the first principle of energy conservation, but fails (in its classical definition) to model systems that express in their behavior the second principle of thermodynamics. Various authors have proposed extensions to the PHS theory, to encompass system which arise from irreversible phenomena.

The framework of **GENERIC** was first developed in the work of ([14], [29], [28]) to encompass both reversible and irreversible physical systems for isolated thermodynamics systems. Later in the work of ([23], [20]) the framework was extended to open systems.

In ([9], [5]) a representation of PHS which are called **pseudo Hamiltonian** was developed to represent a large class of dynamical systems, and furthermore in [6] the framework was used for the study of the stability of dynamical systems.

The papers ([11], [12]) define the **control contact systems**, generalizing the input-output port Hamiltonian systems to cope with models derived from irreversible thermodynamics.

A recent framework to encompass systems which have reversible-irreversible phenomena was developed in [32], namely **Irreversible port-Hamiltonian systems (IPHS)**. These systems express as a structural property the first and second principles of thermodynamics by adding a non linear real function to the dynamics. By definition, IPHS are non-linear systems with a physically meaningful structure and just as PHS systems, they are defined with respect to the total energy stored in the system. This approach makes it possible to interconnect them with other reversible or non-reversible systems, or a combination of both [33]. Some first approaches to control of IPHS have been given in [31] using an IDA-PBC like approach, with the framework of an energy based availability function as a candidate for a Lyapunov function, which is based in the spirit of the works of [1] and [39].

The purpose of this thesis is to extend the passive and non-linear techniques to the control of IPHS, proposing a systematic design method for the control of irreversible port Hamiltonian systems, based on control by interconnection plus damping injection (**Cbi-Di**), altogether with the availability function.

1.2 Organization of the chapters of this thesis

This thesis is divided into five chapters.

- ***Chapter 2***: A brief review of port Hamiltonian systems (PHS) is given along with their main characteristics such as the existence of invariants structures called Casimir functions. The PHS model of a Mass-Spring-Damper system is also shown as an example and Casimir functions for this system are obtained.

  The definition and properties of the Irreversible port Hamiltonian systems (IPHS) are shown, and how in their non-linear structure, they are capable of capturing the second law of thermodynamics; the existence of Casimir functions for this framework is shown. We give the example of a non isothermal RLC system, which can be seen as a coupled

PHS-IPHS and we obtain its Casimir functions. The example of a Continuous Stirred Tank Reactor (CSTR) system is also given, which can be modelled as a pure IPHS.

- ***Chapter 3***: In this chapter, the control by interconnection plus damping injection (Cbi-Di) of PHS systems is reviewed; the Lyapunov stability theorem and the invariance principle are recalled, as they are fundamental to the theory. A Cbi-Di controller is obtained for the Mass-Spring-Damper system as an example.

  Subsequently, the Cbi-Di control is extended to IPHS. A systematic control design method for IPHS is shown, specializing the control by interconnection and damping injection to IPHS.

  The IPHS model of the CSTR is used to synthesize a Cbi-Di controller for a general reaction of m species.

- ***Chapter 4***: The IPHS model of a Gas-Piston system is derived; a Cbi-Di controller is then obtained using the framework of the Cbi-Di control for IPHS.

  The Cbi-Di controller is then simulated using Matlab-Simulink to show the performance of the controller.

- ***Chapter 5***: Conclusions and comments about future work are given in this chapter.

1.3 Main Contributions

The main contributions of this thesis can be summarized in the following points:

- The framework of the control by interconnection plus damping injection (Cbi-Di) is extended to IPHS systems providing a systematic control design method for IPHS.
- The synthesis of an IPHS controller for a CSTR is given and it is shown that it coincides with a controller obtained using an IDA-PBC type of design.
- A Gas-Piston system is used as case study; a Cbi-Di controller is designed and simulations are provided to evaluate different performances.

Chapter 2

REVERSIBLE-IRREVERSIBLE PORT HAMILTONIAN SYSTEMS

This chapter recalls the definition of a port Hamiltonian system, along with its main characteristics and properties. As an example, the PHS model of the classic mechanical Mass-Spring-Damper system is obtained.

In the second part, the definition of an irreversible port Hamiltonian system is presented; its main properties are analyzed, and the main differences with the definition of a PHS system. As an example of the framework, the IPHS models of two systems are shown: a non-isothermal RLC system, which is a system with reversible-irreversible phenomena and a CSTR chemical reactor which is a purely irreversible system.

2.1 Port Hamiltonian Systems - PHS

The framework of port-Hamiltonian system (PHS) arise from the first principle of the thermodynamic and its application to complex multiphysical systems ([36], [37]). The port-Hamiltonian system theory formalizes the basic interconnection laws together with the power preserving energy-storing elements by a geometric structure, using the Hamiltonian function as the total energy of the system ([25], [27], [37], [35]). This approach has been used to model complex and multiphysical system since energy serves as the lingua franca between the systems.

2.1.1 The PHS formulation

Definition 1. *An input/state output/port PHS is defined by the dynamical equation*

$$\begin{aligned} \dot{x}(t) &= [J(x) - R(x)]\frac{\partial H}{\partial x} + g(x)u(t) \\ y(t) &= g^\top(x)\frac{\partial H}{\partial x} \end{aligned} \tag{2.1.1}$$

where $x(t) \in \Re^n$ is the state space vector, $u(t) \in \Re^m$ is the input of the system, $H(x) : \Re^n \to \Re$ is the Hamiltonian energy function and $g(x) \in \Re^{n\times m}$ is the input map. The matrices $J(x) \in \Re^{n\times n}$, $R(x) \in \Re^{n\times n}$ are respectively, the structure matrix and the dissipation matrix of the system which satisfy $J = -J^\top$ and $R = R^\top \geq 0$. The interconnection matrix $J(x)$

by its skew-symmetric property is power conserving. The resistive matrix $R(x)$, by its non negative property express the internal dissipation energy of the system.

The energy balance equation of the function $H(x)$ express the first principle of the thermodynamics. Taking the time derivative of the energy function

$$\begin{aligned}\frac{dH}{dt} &= \frac{dH^\top}{dx}\frac{dx}{dt}\\ &= -\frac{dH^\top}{dx}R\frac{dH}{dx} + \frac{dH^\top}{dx}gu\\ &= -\frac{dH^\top}{dx}R\frac{dH}{dx} + y^\top u \leq u^\top y \end{aligned} \tag{2.1.2}$$

because $R \geq 0$ and where $u^\top y$ is an external supply power; inequality (2.1.2) shows that the PHS cannot store more energy than the one supplied from the external port. Now, consider the following definition.

Definition 2 ([21])**.** *The Poisson bracket is defined with respect to a constant skew-symmetric matrix $J = -J^\top$, acting on any two smooth functions Z, G as*

$$\{Z, G\}_J = \frac{\partial Z^T}{\partial x}(x) J \frac{\partial G}{\partial x}(x) \tag{2.1.3}$$

The structure matrix J not only represents the energy flow between different physical systems domains, but it is also related to a simpletic structure called Poisson bracket (Definition 2) which is also skew symmetric due to the structure matrix J . If J is a Poisson structure, then it satisfies the integrability conditions which are refered as Jacobi identities [37]. The properties of the Poisson bracket implies the existence of energy conservation laws as they define conserved quantities. As we shall see in the next chapter, the energy conservation is fundamental in the passivity based control techniques (PBC).

The dynamical equations of the PHS, with dissipation matrix $R = 0$, can be written in terms of the Poisson bracket as

$$\frac{dx}{dt} = \{x, U\}_J + g(x)u(t)$$

2.1.2 A key property: the Casimir functions

A key property of the PHS framework is the existence of structural invariants. Let us consider a real function C and let us explore the equation

$$\frac{\partial C^\top}{\partial x}(J(x) - R(x)) = 0 \tag{2.1.4}$$

Suppose that equation (2.1.4) has a solution. From the time derivative of C along the trajectories of the port Hamiltonian system (Definition 1), it follows

$$\frac{dC}{dt} = \frac{\partial C^\top}{\partial x}\left[(J - R)\frac{\partial H}{\partial x} + gu\right] = \frac{\partial C^\top}{\partial x}gu \tag{2.1.5}$$

If we set $u = 0$, the function $C(x)$ remains constant along the trajectories of the PHS, with no dependency of the Hamiltonian function H. Furthermore, if $\frac{\partial C^\top}{\partial x}g = 0$ then the invariance holds for any input u. Functions $C(x)$ that satisfy equation (2.1.4) are called Casimir functions of the system and its existence has implications on the stability analysis of a PHS system ([37], [36], [25], [27], [35]). Consider then the following definition.

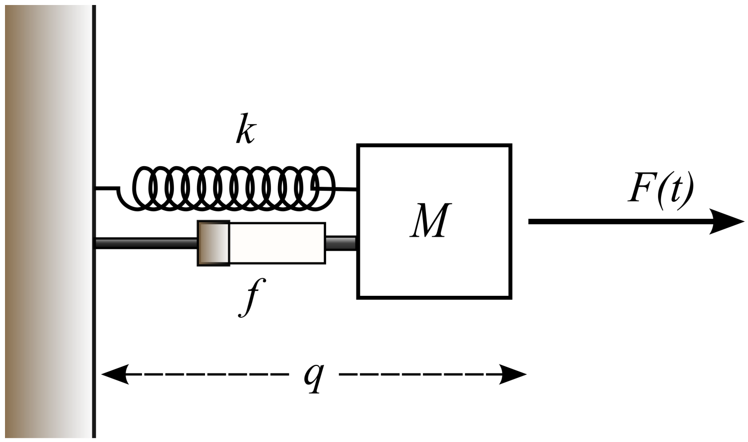

Figure 2.1. Mass Spring Damper system.

Definition 3 ([37])**.** *A Casimir function for an input-output port PHS (Definition 1) is any function $C(x) : \Re^n \to \Re$ which satisfies*

$$\frac{\partial C^\top}{\partial x}(J(x) - R(x)) = 0 \tag{2.1.6}$$

Furthermore, as it was shown in (2.1.5), by setting $u = 0$ then

$$\frac{dC}{dt} = 0$$

Thus, the Casimir function is a conserved quantity of the system for $u = 0$, independently of the Hamiltonian H. If $\frac{\partial C^\top}{\partial x} g = 0$ then the invariance holds for any input u.

The previous results on the Casimir function can be summarized in the next proposition

Proposition 1 ([37])**.** *The function $C(x) : \Re^n \to \Re$ is said to be a Casimir function for the PHS in Definition 1 if and only if*

$$\frac{\partial C^\top}{\partial x} J(x) = 0, \frac{\partial C^\top}{\partial x} R(x) = 0, x \in \Re^n \tag{2.1.7}$$

As we see, the Casimir functions act as invariant structures for a PHS system. This definition is fundamental in PBC techniques which aim at shaping the Hamiltonian energy function. Next we give as an example of the PHS modeling the mass-spring-damper system.

2.1.3 Example: The MSD system

As an illustrative example, we consider the classical benchmark of the mass-spring-damper which is a mechanical system with dissipation. Let us consider Figure 2.1; the PHS formulation of the MSD is done following the Newton's second law:

$$M\ddot{q} = F - kq - f\dot{q}$$

where q is the relative position of the system. The term kq is the force of the spring acting on the mass M which is proportional due to Hooke's law; the term $f\dot{q}$ is the force of the damper acting on the mass. The Hamiltonian energy of the system is then

$$H(q,p) = k\frac{q^2}{2} + \frac{p^2}{2M}$$

Where p is the momentum of the system. The MSD system has the PHS form

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{pmatrix}\dot{q}\\ \dot{p}\end{pmatrix} = \left(\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix} - \begin{bmatrix}0 & 0\\ 0 & f\end{bmatrix}\right)\begin{pmatrix}kq\\ \dot{q}\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}F\\ y &= \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}kq\\ \dot{q}\end{pmatrix} = \dot{q}\end{aligned} \tag{2.1.8}$$

where the structure matrix J of the system, the dissipation matrix and the input map are, respectively

$$J = \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}, \quad R = \begin{bmatrix}0 & 0\\ 0 & f\end{bmatrix}, \quad g = \begin{pmatrix}0\\ 1\end{pmatrix}$$

The structure matrix J satisfies the skew-symmetric property $J = -J^\top$ and the dissipation matrix is such that $R = R^\top \geq 0$ verifying the non-negative condition.

2.1.4 Casimir functions of the MSD system

Let us explore for Casimir functions of the system (2.1.8) by setting $u = 0$. By Proposition 1 we take $C(q,p) : \Re^2 \to \Re$ such that the system of partial differential equations (PDE)

$$\begin{aligned}\frac{\partial C^\top}{\partial x}J(x) &= 0\\ \frac{\partial C^\top}{\partial x}R(x) &= 0\end{aligned}$$

hold for every $x = (q,p) \in \Re^2$; note that $\frac{\partial C^\top}{\partial x} = \begin{pmatrix}\frac{\partial C}{\partial q} & \frac{\partial C}{\partial p}\end{pmatrix}$. The Casimir function then has to satisfy $\frac{\partial C}{\partial q} = \frac{\partial C}{\partial p} = 0$ with solution $C = k \in \Re$ being any real constant; which shows that there are not non trivial Casimir functions for the MSD system.

2.2 Irreversible Port Hamiltonian Systems - IPHS

The irreversible port Hamiltonian systems have been proposed in [32] and [33] as an extension of the port Hamiltonian system framework. IPHS encompass systems arising from the irreversible thermodinamics by expressing as a structural property not only the first thermodynamical principle, which is associated with the energy conservation, but also the second thermodynamical principle, which is associated with the irreversible creation of entropy. In this section, we give the classic IPHS definition proposed in [32]; this definition encompasses systems that express purely irreversible phenomena. Further, the extended IPHS definition ([33]) is given, which encompasses systems with reversible-irreversible phenomena.

2.2.1 The IPHS formulation

Definition 4 ([32]). *An IPHS is defined by the dynamical equations*

$$\begin{aligned}\dot{x} &= J\left(x, \tfrac{\partial U}{\partial x}\right)\frac{\partial U}{\partial x} + g\left(x, \tfrac{\partial U}{\partial x}\right)u \\ y &= g\left(x, \tfrac{\partial U}{\partial x}\right)^\top \frac{\partial U}{\partial x}\end{aligned} \tag{2.2.1}$$

where $x(t) \in \Re^n$ is the state space vector, $u(t) \in \Re^m$ is the input of the system, $U(x) : \Re^n \to \Re$ is the Hamiltonian energy function which is a smooth function of the state x. The structure skew-symmetric matrix is $J = -J^\top$ and the input map is $g \in \Re^{n\times m}$. There exists a smooth entropy function $S(x) : \Re^n \to \Re$. The non-linear modulating function R is defined as

$$R\left(x, \tfrac{\partial U}{\partial x}\right) = \gamma\left(x, \tfrac{\partial U}{\partial x}\right)\{S, U\}_J \tag{2.2.2}$$

where $\gamma\left(x, \frac{\partial U}{\partial x}\right) : \Re^n \to \Re$ is such that $\gamma \geq 0$,i.e, a non linear positive function.

Let us see the balance equations of the energy function $U(x)$ and entropy function $S(x)$ which express respectively, the conservation of the energy and the irreversible creation of entropy. In effect, taking the time derivative of the energy function

$$\begin{aligned}\frac{dU}{dt} &= R\frac{dU^T}{dx}J\frac{dU}{dx} + \frac{dU^T}{dx}gu \\ &= y^\top u\end{aligned}$$

where $\{U, U\}_J = \frac{dU^T}{dx}J\frac{dU}{dx} = 0$ due to the skew-symmetry of the structure matrix J; which express that the IPHS is a lossless dissipative system with supply rate $y^\top u$. Now taking the time derivative of the entropy like function $S(x)$ and setting $u = 0$ for simplicity, we get

$$\begin{aligned}\frac{dS}{dt} &= R\frac{dS^\top}{dx}J\frac{dU}{dx} \\ &= \gamma\left(x, \tfrac{\partial U}{\partial x}\right)\{S, U\}_J^2 = \sigma \geq 0\end{aligned}$$

where σ is the internal entropy production and it shows that the entropy is an increasing function of x and always positive.

As we see in Definition 4, the main difference with the PHS definition lies in the modulating function $R(x)$. In [32],[33] it has been seen that in fact, for thermodynamics systems, J is a matrix whose elements are $1, 0, -1$ and which is associated with the structure of the IPHS. Thus represent the energy flow between different physical systems domains; the modulating function R then captures the dynamical behavior of the system. Definition 4 is useful when one is modeling pure irreversible systems. In the next section we give the example of a CSTR system which can be modelled as an IPHS exploiting Definition 4.

2.2.2 Example: The IPHS model of the CSTR system

In this section we present, as an example, a continuous stirred tank reactor (CSTR), which is a system that can be interpreted within the framework of IPHS.

Let us consider a CSTR system with the following reversible reaction scheme:

$$\sum_{i=1}^{m}\zeta_i A_i \overset{r}{\rightleftharpoons} \sum_{i=1}^{m}\eta_i A_i \tag{2.2.3}$$

with ζ_i, η_i being the constant stoichiometric coefficients for species A_i in the reaction. We will consider the following assumptions for the standard operation of the reactor ([2], [13]):

Assumption 1. *The following holds*

1. *The reactor operates in liquid phase.*
2. *The molar volume of each species are identical and the total volume V in the reactor is constant through the reaction.*
3. *The initial number of moles of a species in the reactor is equal to the number of moles of the inlet of the sames species.*
4. *For a given steady state temperature T and steady state input there is only one possible steady state for the mass. This mean that each steady state temperature is associated with a unique steady state temperature.*

The IPHS model of the CSTR is [31].

$$\dot{x}(t) = RJ\frac{\partial U}{\partial x}(x) + gu(t)$$

with the state vector $x = \begin{bmatrix} \mathbf{n} & S \end{bmatrix}^T$, where $\mathbf{n} = (n_1, ..., n_m)^T$ with n_i the number of moles of the species i inside the reactor; $S(x)$ the total entropy of the system and $U(x)$ the internal energy function, and

$$J = \begin{bmatrix} 0 & \cdots & 0 & \bar{\nu}_1 \\ & & & \vdots \\ 0 & \cdots & 0 & \vdots \\ 0 & \cdots & 0 & \bar{\nu}_m \\ -\bar{\nu}_1 & \cdots & -\bar{\nu}_m & 0 \end{bmatrix}, \quad \frac{\partial U}{\partial x} = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_m \\ T \end{bmatrix}$$

where J is a constant skew-symmetric matrix whose elements are the signed stoichiometric coefficients of the chemical reaction $\bar{\nu}_i = \zeta_i - \eta_i$, a number which is positive or negative depending on whether the species i is a product or a reactant; $\frac{\partial U}{\partial x}$ corresponds to the intensive variables with T being the temperature in the reactor and μ_i the chemical potential of the species i; R is the modulating function and is given by

$$R = \frac{rV}{T}$$

where $r = r(\mathbf{n}, T)$ is the reaction rate which depends on the temperature and on the reactant mole numbers vector $\mathbf{n}$. The input vector is $u = [u_1, u_2]^T$ with $u_1 = F/V$ the dilution rate, where F is the volumetric flow rate, and $u_2 = Q$ the heat flux from the cooling jacket; the input map g is given by

$$g = \begin{bmatrix} \bar{\mathbf{n}} & \mathbf{0} \\ \phi(x) & 1/T \end{bmatrix}$$

with $\bar{\mathbf{n}} = \mathbf{n}_e - \mathbf{n}$, where $\mathbf{n}_e = (n_{e1}, ..., n_{em})^T$ is the vector containing the numbers of moles of species i at the inlet and $\phi(x) = \sum_{i=1}^{m}(n_{ei}s_{ei} - n_i s_i) + \frac{n_{ei}}{T}(h_{ei} - Ts_{ei} - \mu_i)$, where s_{ei} is the inlet molar entropy,s_i is the molar entropy and h_{ei} is the inlet specific molar enthalpy of species i.

In the next section a more general definition which encompass systems with reversible-irreversible phenomena shall be shown.

2.2.3 The Coupled PHS-IPHS formulation

The coupled PHS-IPHS system formulation has been defined in [33] to encompass systems that express in its structure reversible-irreversible phenomena. They retained much of the properties of the IPHS formulation as we shall see, expressing in its structure the first and second thermodynamics laws. Let us define the coupled PHS-IPHS definition.

Definition 5 ([33]). *A coupled PHS-IPHS is defined by the dynamical equation*

$$\begin{aligned} \dot{x} &= J_{ir}\left(x, \tfrac{\partial U}{\partial x}\right) \frac{\partial U}{\partial x} + g\left(x, \tfrac{\partial U}{\partial x}\right) u \\ y &= g\left(x, \tfrac{\partial U}{\partial x}\right)^{\top} \frac{\partial U}{\partial x} \end{aligned} \tag{2.2.4}$$

where $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ the input, the smooth function $U(x) : \Re^n \to \Re$ is the Hamiltonian and $g \in \Re^{n\times m}$ is the input map. The difference with Definition 4 lies in the skew-symmetric structure matrix $J_{ir} \in \Re^{n\times n}$ which is defined as

$$J_{ir}\left(x, \tfrac{\partial U}{\partial x}\right) = J_0(x) + R\left(x, \tfrac{\partial U}{\partial x}\right) J \tag{2.2.5}$$

with $J = -J^T$, $J_0 = -J_0^T$ and there exists a smooth entropy like function $S(x) : \Re^n \to \Re$ which is a Casimir function of J_0, i.e.,

$$\frac{\partial S}{\partial x}^{\top} J_0 = 0. \tag{2.2.6}$$

The non-linear modulating function R is defined as

$$R\left(x, \tfrac{\partial U}{\partial x}\right) = \gamma\left(x, \tfrac{\partial U}{\partial x}\right) \{S, U\}_J$$

where $\gamma\left(x, \frac{\partial U}{\partial x}\right) : \Re^n \to \Re$ is a non linear positive function.

The balance equations of the entropy function $S(x)$ and the energy function $U(x)$ goes similar to Definition 4. Taking the time derivative of the energy function gives

$$\begin{aligned} \frac{dU}{dt} &= \frac{dU^T}{dx}(J_0 + RJ)\frac{dU}{dx} + \frac{dU^T}{dx} gu \\ &= y^T u \end{aligned}$$

where $\{U, U\}_{J_{ir}} = \frac{dU^T}{dx}(J_0 + RJ)\frac{dU}{dx} = 0$ by skew-symmetry of J_{ir}, expressing that the coupled PHS-IPHS is a lossless dissipative system with supply rate $y^T u$. By setting $u = 0$ and taking the time derivative of the entropy function, it follows that

$$\begin{aligned} \frac{dS}{dt} &= \frac{dS^T}{dx} J_0 \frac{dU}{dx} + R\frac{dS^T}{dx} J \frac{dU}{dx} \\ &= \{S, U\}_{J_0} + \gamma\left(x, \tfrac{\partial U}{\partial x}\right) \{S, U\}_J^2 \\ &= \gamma\left(x, \tfrac{\partial U}{\partial x}\right) \{S, U\}_J^2 = \sigma \geq 0 \end{aligned}$$

where the term $\{S, U\}_{J_0} = 0$ because of (2.2.6) and where σ expresses the internal entropy production. The coupled PHS-IPHS formulation expresses the first and second principle, and encompasses systems which have a reversible-irreversible phenomena.

2.2.4 Casimir functions for Coupled PHS-IPHS

As in port Hamiltonian systems, one can look for Casimir functions for a coupled PHS-IPHS. Let us take C a real function of the states of the systems and suppose that the following relation holds

$$\frac{\partial C^\top}{\partial x} J_{ir} = 0 \tag{2.2.7}$$

If we take the time derivative of C, with the condition (2.2.7), it follows that

$$\frac{dC}{dt} = \frac{\partial C^\top}{\partial x}\left[J_{ir}\frac{\partial U}{\partial x} + gu\right] = \frac{\partial C^\top}{\partial x} gu \tag{2.2.8}$$

If $u = 0$ then (2.2.8) remains true for every $C(x)$ along the trajectories of the PHS-IPHS, independently of the Hamiltonian $U(x)$.

Definition 6. *A Casimir function for a coupled PHS-IPHS (Definition 5) is any function* $C(x) : \Re^n \to \Re$ *which satisfies*

$$\frac{\partial C^\top}{\partial x} J_{ir} = 0 \tag{2.2.9}$$

Furthermore, as it was shown in (2.2.8), by setting $u = 0$ *then*

$$\frac{dC}{dt} = 0$$

Thus the Casimir function is a conserved quantity of the system for $u = 0$*, independently of the Hamiltonian* U*. If* $\frac{\partial C^\top}{\partial x} g = 0$ *then the invariance holds for any input* u*.*

Proposition 2. *Let* $C(x) : \Re^n \to \Re$ *be a Casimir function for the PHS-IPHS in Definition 5 if and only if*

$$\frac{\partial C^\top}{\partial x} J_{ir} = 0 \tag{2.2.10}$$

where $J_{ir} = -J_{ir}^\top = J_0 + RJ$ *is the structure matrix of the PHS-IPHS.*

Note that as J_0 and J are skew-symmetric, it does not necessarily follow that $\frac{\partial C^\top}{\partial x} J_0 = 0$ and $\frac{\partial C^\top}{\partial x} J = 0$.

The following section studies the example of a non-isothermal RLC system which is a system than can be interpreted within the framework of the coupled PHS-IPHS.

2.2.5 Example: non-isothermal RLC system

Consider a RLC system connected in series including the dynamics of the thermal effects of its electrical components. So we can consider that all electrical components, i.e, the resistor $r(S)$; the inductor $L(S)$ and the capacitor $C(S)$ are a function of the temperature and therefore of the entropy of the system.

The internal energy of the system $U(Q, \phi, S)$ shall be the sum of: the energy of the capacitor; the energy of the inductor and some thermal related energy function $U_s(S)$ associated to the components of the system, with Q being the charge of the capacitor; ϕ being the flux of the inductor and S the entropy of the system.

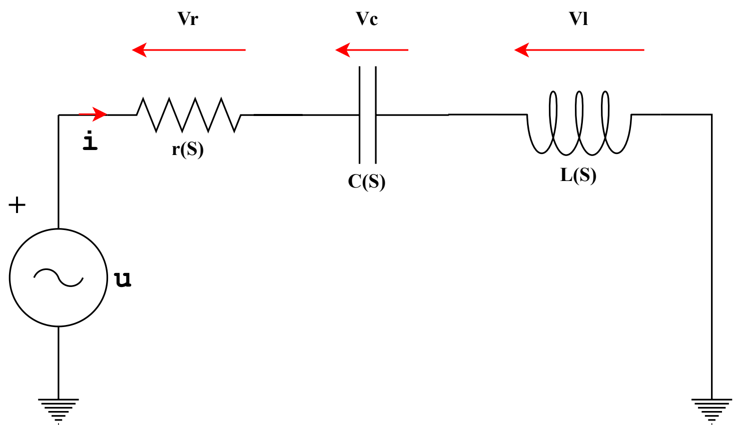

Figure 2.2. RLC circuit where each component includes the dynamics of the thermal effects.

The energy then can be written as

$$U(Q,\phi,S) = \frac{1}{2}\frac{Q^2}{C(S)} + \frac{1}{2}\frac{\phi^2}{L(S)} + U_s(S) \tag{2.2.11}$$

where the time variation of the internal energy is

$$\frac{dU}{dt} = \frac{\partial U}{\partial Q}\dot{Q} + \frac{\partial U}{\partial \phi}\dot{\phi} + \frac{\partial U}{\partial S}\dot{S} \tag{2.2.12}$$

We denote i_x, V_x with $x = r, l, c$ as the current and voltage of the resistor, inductor and capacitor, respectively. If we apply Kirchhoff's laws then it is clear that

$$\begin{aligned} i_r &= i_l = i_c = i \\ u &= V_r + V_l + V_c \end{aligned}$$

For each component the following laws hold

$$\begin{aligned} V_r &= ir & V_l &= -L\frac{di}{dt} & i &= C\frac{dV_c}{dt} \\ \phi &= Li & Q &= CV_c \end{aligned}$$

From this set of equations we have that $\dot{Q} = \frac{\phi}{L}$; the Kirchhoff law for the voltage of the system gives the behavior of the flux as $\dot{\phi} = -\frac{Q}{C} - r\frac{\phi}{L} + u$ where u is the input of the system which is the voltage source. The dynamical equation for the entropy S of the system follows from equation (2.2.12); let us expand the equation and note that

$$\frac{dU}{dt} = -r(S)\left(\frac{\phi}{L(S)}\right)^2 + \frac{\partial U}{\partial S}\frac{dS}{dt} + y_e^T u \tag{2.2.13}$$

From Gibb's relation [4] it is known that $\frac{\partial U}{\partial S} = T(S)$. Taking $u = 0$ it follows that $\dot{U} = 0$, as the internal energy variation must be zero, then it goes that

$$\frac{dS}{dt} = \frac{r(S)}{T(S)}\left(\frac{\phi}{L(S)}\right)^2 = \sigma_r. \tag{2.2.14}$$

The term σ_r corresponds to the internal entropy production of the system. The PHS-IPHS formulation of the thermodynamic RLC circuit is then

$$\begin{bmatrix} \dot{Q} \\ \dot{\phi} \\ \dot{S} \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \frac{r}{T}\frac{\phi}{L} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \right) \begin{bmatrix} \frac{Q}{C} \\ \frac{\phi}{L} \\ T \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \tag{2.2.15}$$

where

$$J_0 = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad R = \frac{r}{T}\frac{\phi}{L}$$

Note that the RLC system (2.2.15) has the structure of the Definition 5 with a structure matrix composed of an irreversible part related to the dissipation and a reversible part related to Kirchhoff's law.

2.2.6 Casimir functions of the non-isothermal RLC system

Now, we shall study if the non-isothermal RLC system (2.2.15) has useful Casimir functions. By proposition 2, we have to look for a function $C(x) : \Re^3 \to \Re$ such that

$$\begin{bmatrix} \frac{\partial C}{\partial Q} & \frac{\partial C}{\partial \phi} & \frac{\partial C}{\partial S} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -\frac{r}{T}\frac{\phi}{L} \\ 0 & \frac{r}{T}\frac{\phi}{L} & 0 \end{bmatrix} = 0$$

It is easy to note that the system is reduced to

$$\begin{aligned} \frac{\partial C}{\partial \phi} &= 0 \\ \frac{\partial C}{\partial Q} &= -\frac{r}{T}\frac{\phi}{L}\frac{\partial C}{\partial S} \end{aligned}$$

As the first equation imposes that the Casimir has to have no dependency of the flux ϕ of the system, there isn't a non trivial solution that solves the PDE.

Even though the Casimir functions of a system when we have no input are usually trivial, we shall see in Chapter 3 that they are proven to be a powerful tool in the energy-Casimir approach when a control input u is considered.

Chapter 3

PASSIVITY BASED CONTROL METHODS APPLIED TO IPHS

In the present chapter the main contributions of this thesis are shown. The framework of the Passivity Based Control (PBC) with emphasis on the control by interconnection plus damping injection approach (Cbi-Di) is presented.

We first recall the basics of the formulation applied to PHS. In order to do that, some definitions about Lyapunov stability and Lasalle's invariance principle are shown. As an illustrative example, a Cbi-Di controller for the mass-spring-damper system (2.1.8) is obtained. Subsequently, we extend the Cbi-Di framework to the control of IPHS; as we shall see, Casimir functions shall be instrumental in the design. The notion of the availability function shall complement the control design as it provides candidates for Lyapunov functions for the irreversible part of the IPHS.

The example of the CSTR system, whose IPHS model has been obtained in section (2.2.2), is used later to design a controller within the framework of the Cbi-Di controller for PHS-IPHS.

3.1 Passivity Based Control of PHS

The Passivity based control framework has been used to model electrical, mechanical and complex physical systems since the control design has a physical interpretation. The PBC framework aims at rendering the closed-loop Hamiltonian function, which has been shown to serve as a candidate for a Lyapunov function, to some desired and useful energy function, which has a new a desired equilibrium dynamic. It provides a systematic framework to achieve stabilization and asymptotic stability for a PHS, and has been used successfully for control design ([25], [27], [37]).

In this section we give the standard definition of the Cbi-Di control for PHS; some definitions concerning Lyapunov theorem and the Lasalle's invariance principle are shown as they are fundamental in the Cbi-Di approach to ensure stability and asymptotic stability of the closed-loop system.

3.1.1 Lyapunov Stability Theorem

The Lyapunov stability analysis formalizes the idea that all systems shall tend to a minimum energy state. It gives a powerful method in stability analysis of **non-linear** systems and in the passivity based control framework for PHS it is fundamental, as the energy functions are candidates for Lyapunov functions.

Theorem 1 ([3]). *Let D be a compact subset of the state space of a system, containing the equilibrium point x_0, and let there be a function $V : D \to \Re$. The equilibrium point x_0 is stable (in the sense of Lyapunov) if V satisfies the following conditions:*

1. *$V(x) \geq 0$, for all $x \in D$*
2. *$V(x) = 0$ if and only if $x = x_0$*
3. *For all $x(t) \in D$,*

$$\frac{dV(t)}{dt} = \frac{\partial V(t)}{\partial x}\frac{\partial x(t)}{dt} \leq 0$$

*Furthermore, if $\dot{V}(x(t))$ is strictly negative, i.e $\dot{V}(x(t)) < 0$, then the equilibrium is said to be **asymptotically stable***

3.1.2 Lasalle's Invariance Principle

The Lyapunov stability theorem guarantees asymptotic stability of the system if we can find a Lyapunov function that is strictly decreasing away from the equilibrium point, as is stated in Theorem 1; but the strictly negative derivative condition on Theorem 1 can be relaxed while ensuring system asymptotic stability. First, we give a definition concerning to an invariant manifold and a positively invariant set. Then we show the Lasalle's invariance principle.

Definition 7 ([15]). *Let $\Omega \in \Re^m \times R^n$. The set Ω is said to be an invariant manifold if*

$$(x(0), \xi(0)) \in \Omega \Leftrightarrow (x(t), \xi(t)) \in \Omega, \forall t \geq 0 \tag{3.1.1}$$

Given $t = t_0$. A set Ω is set to be positively invariant if $x(t_0) \in \Omega$, then $x(t) \in \Omega$ for all $t \geq t_0$.

For example, the multi level set $\Omega_\kappa = \{(x, \xi) \in \Re^n \times \Re^m \mid \xi = F(x) + \kappa\}$ is an invariant manifold, where κ is a vector of constants. Next, we define the Lasalle's invariance principle.

Theorem 2 ([15]). *Consider the non-linear dynamical system $\dot{x}(t) = f(x(t))$ with $x(0) = x_0$. Assume that $D_c \subset D$ is a compact positively invariant set with respect to the non-linear system, and assume there exits a continuously differentiable function $V : D_c \to \Re$ such that $\dot{V}(x)f(x) \leq 0$, $x \in D_c$. Let $\Omega \doteq \left\{x \in D_c : \dot{V}(x)f(x) = 0\right\}$ and let $\mathcal{M}$ be the largest invariant set contained in Ω. If $x(0) \in D_c$, then $x(t) \to \mathcal{M}$ as $t \to \infty$.*

3.1.3 Control by Interconnection of PHS

The paradigm of control by interconnection has been used within the framework of passivity based control techniques for stabilization. Control by interconnection aims at shaping the Hamiltonian function and the structure matrices by state feedback with a controller, which is also a PHS system; the closed-loop system has proven to be also a PHS. The next step is to find useful Casimir functions for this new PHS, as they allow to move the equilibrium point of the Hamiltonian energy function; this is done by solving a set of PDE [37].

We shall follow figure 3.1 for the control design; the first loop considers the control by interconnection (Cbi) part, which is in charge of the stability of the system, while the second

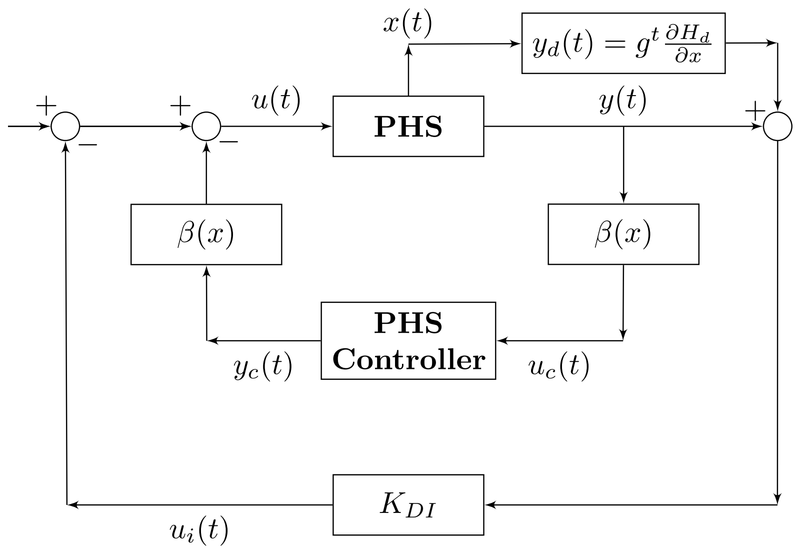

Figure 3.1. Control by interconnection plus damping injection control of a PHS.

loop considers the injection of damping (Di), which is in charge of the asymptotic stability at the equilibrium.

Let us consider a PHS controller in the form

$$\begin{aligned} \dot{\xi}(t) &= [J_c(\xi) - R_c(\xi)]\,\frac{\partial H_c}{\partial \xi} + g_c(\xi)u_c(t) \\ y_c(t) &= g_c^\top \frac{\partial H_c}{\partial \xi} \end{aligned} \tag{3.1.2}$$

with $\xi \in \Re^{n_c}$ the state space vector of the system, $y_c, u_c \in \Re^{m_c}$ the output and input of the system, respectively; a Hamiltonian smooth energy function of ξ, $H_c : \Re^{n_c} \to \Re$ with $g_c \in \Re^{n_c \times m_c}$ the input map. The structure matrix of the system J_c is such that $J_c = -J_c^\top$ and the dissipation matrix R_c satisfies $R_c = R_c^\top \geq 0$.

The interconnection of the PHS controller with the PHS system is done via the modulated negative feedback, with $\beta \in \Re$

$$\begin{pmatrix} u \\ u_c \end{pmatrix} = \begin{pmatrix} 0 & -\beta \\ \beta & 0 \end{pmatrix} \begin{pmatrix} y \\ y_c \end{pmatrix} \tag{3.1.3}$$

Taking a standard PHS system (2.1.1), then the closed-loop system can be written as

$$\begin{aligned} \underbrace{\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix}}_{x_d} &= \left(\underbrace{\begin{bmatrix} J & -g\beta g_c^\top \\ g_c \beta g^\top & J_c \end{bmatrix}}_{J_d} - \underbrace{\begin{bmatrix} R & 0 \\ 0 & R_c \end{bmatrix}}_{R_d} \right) \begin{pmatrix} \frac{\partial H_d(x,\xi)}{\partial x} \\ \frac{\partial H_d(x,\xi)}{\partial \xi} \end{pmatrix} + \underbrace{\begin{pmatrix} g \\ \mathbf{0} \end{pmatrix}}_{g_d} u_i \\ y_d &= \begin{pmatrix} g^\top & \mathbf{0} \end{pmatrix} \begin{pmatrix} \frac{\partial H_d(x,\xi)}{\partial x} \\ \frac{\partial H_d(x,\xi)}{\partial \xi} \end{pmatrix} \end{aligned} \tag{3.1.4}$$

which is a PHS system with structure matrix $J_d = -J_d^\top$, dissipation matrix $R_d = R_d^\top \geq 0$, both matrices of order $(n{+}n_c)\times(n{+}n_c)$ and the input map g_d with order $(n{+}n_c)\times(m{+}m_c)$, where $\mathbf{0}$ is a null matrix of order $n_c \times m_c$; the closed-loop Hamiltonian energy function is $H_d = H + H_c$.

The next step is to find structural invariant functions. The Casimir functions can be restricted, without loss of generality, to the set of functions [37]

$$C_i(x,\xi_i) = F_i(x) - \xi_i,\ i = 1,..,l \leq n_c \tag{3.1.5}$$

where C_i is the Casimir function associated to the state ξ_i of the controller and $F(x) = [F_1,...,F_l] \in \Re^l$ is a collection of smooth functions F_i of x. If the Casimir function exists, the relation $\xi - F(x) = \kappa$ with $\kappa = [\kappa_1,...,\kappa_l] \in \Re^l$ a vector of constants that depend on the initial states of the plant and the controller, holds on every invariant set $\Omega = \{(x,\xi) \in \Re^{x\times\xi} \mid C(x,\xi) = -\kappa\}$. The closed-loop Hamiltonian energy candidate to a Lyapunov function, can be rewritten as a function of the states of the PHS system as $H_d(x) = H(x) + H_c(F(x)+\kappa)$, and the shaping control input as the negative feedback

$$u_e = -y_c\,(F(x)+\kappa) = -g_c^\top \frac{\partial H_c}{\partial \xi} \tag{3.1.6}$$

The Casimir functions are invariants of the structure of the closed-loop system (3.1.4), which means that the relation $\frac{\partial C^\top}{\partial x_d} J_d = 0$ is satisfied. This condition leads to the set of partial differential equations known as **matching equations**

$$\begin{aligned} \frac{\partial F^\top}{\partial x} J \frac{\partial F}{\partial x} &= J_c \\ R\frac{\partial F}{\partial x} &= 0 \\ R_c &= 0 \\ \frac{\partial F^\top}{\partial x} J &= g_c \beta g^\top \end{aligned} \tag{3.1.7}$$

The third equation in (3.1.7) is known as the *dissipation obstacle* ([37], [38]) since it dictates that the variables of the system that has dissipation cannot be shaped. Assuming that such F smooth function exists then the control action (3.1.6) shapes the Hamiltonian energy of the system, and the function $H_d(x) = H(x) + H_c(F(x)+\kappa)$ is a Lyapunov function candidate for the closed-loop system and the system

$$\begin{aligned} \dot{x} &= [J - R]\frac{\partial H_d}{\partial x} + g u_i \\ y_d &= g^\top \frac{\partial H_d}{\partial x} \end{aligned}$$

is stable with respect to some desire equilibrium point x^* for a particular choice of ξ^*.

3.1.4 Damping injection

The energy shaping control action renders the Hamiltonian energy function into a Lyapunov function candidate for the system with a strict minimum in some desired equilibrium point x^* for a particular choice of ξ^*. The damping injection control action renders the system asymptotically stable. Taking x^* as the minimum of the Hamiltonian function, and setting the control input

$$u_i = -Ky_d = -Kg^\top \frac{\partial H_d}{\partial x} \tag{3.1.8}$$

where $K = K^T \geq 0$ is a symmetric semi positive matrix, guarantees asymptotic stability for the closed-loop system at the point (x^*, ξ^*) by the application of Lasalle's invariance principle. In fact, with the control action (3.1.8) and setting $M = gKg^T \geq 0$ the closed-loop system can be written as

$$\dot{x}(t) = (J - M)\frac{\partial H_d}{\partial x}$$

Taking the time derivative of the closed-loop system we obtain

$$\begin{aligned}
\frac{dH_d}{dt} &= \frac{\partial H_d^\top}{\partial x}(J - M)\frac{\partial H_d}{\partial x} \\
&= \{H_d, H_d\}_J - \{H_d, H_d\}_M \\
&= -\{H_d, H_d\}_M < 0
\end{aligned}$$

Since H_d is a Casimir function of J it follows that $\{H_d, H_d\}_J = 0$. By Lasalle's invariance principle (Theorem 2, section 3.1.2) then the closed-loop system converges asymptotically to x^*.

3.1.5 Example: Cbi-Di for the MSD system

In this section we design, as an example of the framework, a Cbi-Di controller for the MSD system analyzed in section 2.1.3. Since the system has dissipation in the p coordinate, we aim at rendering the closed-loop system to a point $x^* = (q^*, 0)$. Consider the PHS controller

$$\dot{\xi} = (J_c - R_c)\frac{\partial H_c}{\partial \xi} + g_c u_c$$

We look for a Casimir function of the form $C(q,p) = F - \xi$ such that $\xi = F + \kappa$, with $F \in \Re$ a smooth function of the states of the system and $\kappa \in \Re$ a constant that depends on the initial states of the controller and the system. For simplicity, we take $\beta(x) = 1$. The gradient F with respect to the states of the system takes the form

$$\frac{\partial F^\top}{\partial x} = \begin{pmatrix} \frac{\partial F}{\partial q} & \frac{\partial F}{\partial p} \end{pmatrix}$$

The first, second and fourth equation of (3.1.7) give, respectively

$$J_c = 0, \quad \frac{\partial F}{\partial p} = 0, \quad \frac{\partial F}{\partial x} = g_c$$

where $\frac{\partial F}{\partial p}$ was expected to be null due to the **dissipation obstacle** on the coordinate of the state. By taking $g_c = 1$ for simplicity, and by simple integration $F(q,p) = F(q) = q$, where it is easy to note that $\xi = q$. We choose the following Hamiltonian as a candidate Lyapunov energy function

$$H_d = (k + k_0)\frac{(q - q^*)^2}{2} + \frac{p^2}{2M}$$

where the energy of the controller is such that $H_c = H_d - H$, where we recall that $H = k\frac{q^2}{2} + \frac{p^2}{2M}$. The controller energy function is then chosen as

$$H_c = k_0\frac{q^2}{2} - (k+k_0)qq^* + \frac{(k+k_0)}{2}(q^*)^2 = k_0\frac{\xi^2}{2} - (k+k_0)\xi q^* + \frac{(k+k_0)}{2}(q^*)^2$$

The PHS controller can then be expressed as

$$\begin{aligned} \dot{\xi} &= u_c \\ y_c &= k_0\xi - (k+k_0)q^* \end{aligned} \tag{3.1.9}$$

which is a controller with integral-proportional action. The energy shaping control action is given by

$$u_e = -g_c^\top \frac{\partial H_c}{\partial \xi} = (k+k_0)q^* - k_0\xi$$

For the damping injection control action, take $K \in \Re$ such that $M = gKg^\top \in \Re^{2\times 2} \geq 0$. A possible choice for K is to take $K = \alpha \geq 0$ a tuning parameter which gives

$$M = \begin{bmatrix} 0 & 0 \\ 0 & \alpha \end{bmatrix}$$

with M being a semi positive matrix. The damping injection control action can be obtained as

$$u_i = -Kg^\top \frac{\partial H_d}{\partial x} = -\alpha \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} (k+k_0)(q-q^*) \\ \frac{p}{m} \end{bmatrix} = -\alpha\frac{p}{m}$$

The closed-loop system, with the addition of damping, can be written as

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & \alpha \end{bmatrix} \right) \begin{bmatrix} (k+k_0)(q-q^*) \\ \frac{p}{M} \end{bmatrix}$$

3.2 Passivity Based Control of IPHS

In this section the main contributions of this thesis are shown with the synthesis of a control by interconnection plus damping injection (Cbi-Di) controller framework for the control of IPHS. As IPHS retain much of the PHS, PBC techniques such as Cbi-Di can be further explored to the control of IPHS. Thus we synthesize a Cbi-Di controller through a systematic design. We shall exploit Definition 5 which encompass systems with reversible-irreversible phenomena. This definition shows that an IPHS system can be seen as a composition of a conservative part and an irreversible part. Furthermore, we shall exploit Definition 8 which is the availability function of an irreversible process and serves as candidate Lyapunov function.

The control by interconnection plus damping injection is done following Figure 3.2. The IPHS system is interconnected with an IPHS controller in the first loop through a state space modulated function, and is in charge of placing the closed-loop equilibrium point. The controller is used to render the closed-loop Hamiltonian energy function such that it has a minimum at the desire equilibrium point and is now a candidate Lyapunov function by the use of Definition 8. A second loop, with a damping injection control action is design to ensure asymptotic stability of the closed-loop system. The damping injection control input is performed with respect to the closed-loop system output which is the conjugated output to the closed-loop Hamiltonian function.

The final control input takes the form $u = u_e + u_i$ where u_e is the input due to the energy shaping action and u_i is the input due to the damping injection action.

3.2.1 Control by interconnection of IPHS

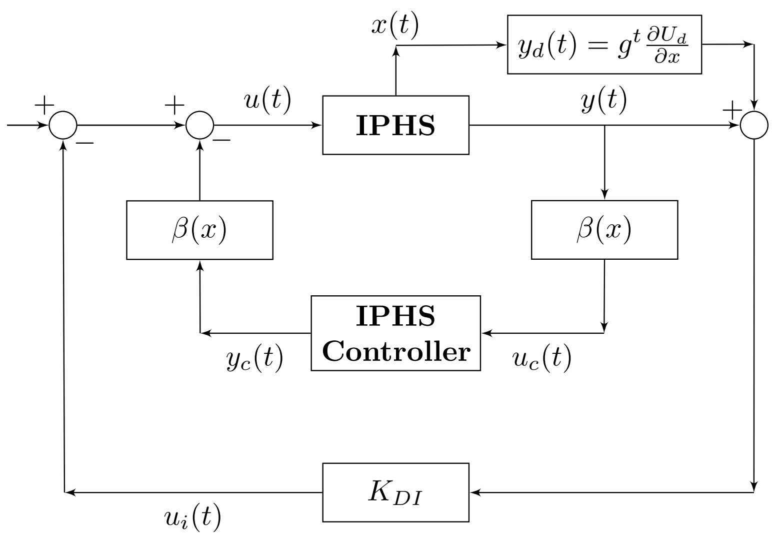

Figure 3.2. Control by interconnection plus damping injection control of an IPHS.

Let us consider the IPHS controller

$$\begin{aligned} \dot{\xi} &= \bar{R}\left(\xi, \tfrac{\partial U_c}{\partial \xi}\right) J_c \frac{\partial U_c}{\partial \xi}(\xi) + g_c\left(\xi, \tfrac{\partial U_c}{\partial \xi}\right) u_c(t) \\ y_c &= g_c^T\left(\xi, \tfrac{\partial U_c}{\partial \xi}\right) \frac{\partial U_c}{\partial \xi}(\xi) \end{aligned} \tag{3.2.1}$$

with $\xi \in \Re^{n_c}$ the state space vector $y_c, u_c \in \Re^{m_c}$ the output and input of the system, respectively. The mapping $g_c(\xi) \in \Re^{n_c \times m_c}$, a Hamiltonian smooth function $U_c(\xi)$ and $\bar{R}\left(\xi, \frac{\partial U_c}{\partial \xi}\right)$ a modulating non-linear function. The interconnection between the states is via the modulated power-preserving interconnection

$$\begin{pmatrix} u_e \\ u_c \end{pmatrix} = \begin{pmatrix} 0 & -\beta(x) \\ \beta(x) & 0 \end{pmatrix} \begin{pmatrix} y \\ y_c \end{pmatrix} \tag{3.2.2}$$

where $\beta(x) \in \Re$. The closed-loop system, following the first loop of figure 3.2, between a standard IPHS (2.2.4) and the IPHS controller (3.2.1) takes the form

$$\begin{pmatrix} \dot{x} \\ \dot{\xi} \end{pmatrix} = \underbrace{\begin{pmatrix} J_{ir} & -g\beta g_c^T \\ g_c \beta g^T & \bar{R} J_c \end{pmatrix}}_{J_d} \begin{pmatrix} \frac{\partial U_d(x,\xi)}{\partial x} \\ \frac{\partial U_d(x,\xi)}{\partial \xi} \end{pmatrix} + \begin{pmatrix} g \\ \mathbf{0} \end{pmatrix} u_i \tag{3.2.3}$$

with $\mathbf{0}$ a null matrix of appropriate dimensions and a closed-loop Hamiltonian function $U_d(x,\xi) = U(x) + U_c(\xi)$. We look for structural invariant functions of the form $C_i(x,\xi_i) = F_i(x) - \xi_i$, $i = 1,..,l$ where $F(x) = [F_1,...,F_l] \in \Re^l$ is a collection of smooth well defined functions F_i of x. Assuming that these invariant functions exist, then on every invariant manifold the relation $\xi - F(x) = \kappa$ with $\kappa = [\kappa_1,...,\kappa_l] \in \Re^l$ holds, where κ is a collection of constants that depends on the initial states of the system and the controller. The closed-loop Hamiltonian energy function can then be expressed in terms of the states of the plant $U_d(x) = U(x) + U_c(F(x)+\kappa)$, and the energy shaping control action as the negative state feedback

$$u_e = -\beta(x) g_c^\top \frac{\partial U_c}{\partial \xi} \tag{3.2.4}$$

The Casimir functions, which are invariants of the structure of the system, satisfy the invariant relation $\frac{\partial C^\top}{\partial x} J_d = 0$, with

$$J_d = \begin{pmatrix} J_{ir} & -g\beta g_c^T \\ g_c \beta g^T & \bar{R} J_c \end{pmatrix}$$

The invariant condition gives

$$\begin{pmatrix} \frac{\partial F^T}{\partial x}(x) & -I \end{pmatrix} \begin{pmatrix} J_{ir} & -g\beta g_c^T \\ g_c \beta(x) g^T & \bar{R} J_c \end{pmatrix} = 0$$

where I is an identity matrix of proper dimension. This condition leads to the following set of PDE

$$\begin{aligned} \frac{\partial F^T}{\partial x}(x) J_{ir} &= g_c \beta g^T \\ -\frac{\partial F^T}{\partial x}(x) g \beta g_c^T &= \bar{R} J_c \end{aligned} \tag{3.2.5}$$

Taking transpose in the first equation in (3.2.5), we get $J_{ir}^T \frac{\partial F}{\partial x} = g\beta^T g_c^T$ but $J_{ir} = -J_{ir}^T$ then

$$-J_{ir} \frac{\partial F}{\partial x} = g\beta^T g_c^T = g\beta g_c^T \tag{3.2.6}$$

Replacing equation (3.2.6) on the left hand side of the second equation in (3.2.5) it yields

$$\frac{\partial F^T}{\partial x} J_{ir} \frac{\partial F}{\partial x} = \bar{R} J_c \tag{3.2.7}$$

The left hand side of (3.2.7) is a skew symmetric matrix so the right hand side should also be a skew symmetric matrix, which is satisfied as J_c is skew-symmetric by definition. Summarizing the results, we get the system of PDE

$$\begin{aligned} \frac{\partial F^T}{\partial x}(x) J_{ir} &= g_c \beta g^T \\ \frac{\partial F^T}{\partial x} J_{ir} \frac{\partial F}{\partial x} &= \bar{R} J_c \end{aligned} \tag{3.2.8}$$

These are the matching equations for an IPHS with a controller in IPHS form, using the state modulated interconnection (3.2.2). These matching equations are analogous to the case of control by interconnection of PHS with the difference that J_{ir} depends on the modulating functions R and that the control structure J_c includes a modulating function $\bar{R}$. Assuming that the smooth function $F(x)$ exists, the control law (3.2.4) shapes the closed-loop Hamiltonian function as $U_d(x) = U(x) + U_c(F(x+\kappa))$. Furthermore, the energy-input allows to interpret the closed-loop system as an IPHS. In effect, notice that

$$\frac{dx}{dt} = RJ_{ir}\frac{\partial U}{\partial x} - g\,\underbrace{\beta g_c \frac{\partial (U_c \circ F)}{\partial \xi}}_{u_e} \tag{3.2.9}$$

Using the first equation in (3.2.8) and the skew-symmetric property of J_{ir}, the relation (3.2.9) can be rewritten as

$$\begin{aligned}\frac{dx}{dt} &= RJ_{ir}\frac{\partial U}{\partial x} + J_{ir}\frac{\partial F}{\partial x}\frac{\partial (U_c \circ F)}{\partial \xi}\\ &= RJ_{ir}\frac{\partial U}{\partial x} + J_{ir}\frac{\partial U_c}{\partial x}\end{aligned}$$

Finally, by simple factorization and adding an input u_i to the closed-loop system, we get

$$\begin{aligned}\dot{x} &= J_{ir}\frac{\partial U_d}{\partial x} + gu_i\\ y_d &= g^\top \frac{\partial U_d}{\partial x}\end{aligned} \tag{3.2.10}$$

where y_d is the passive output defined with respect to $U_d(x)$. This approach allows to see the closed-loop system as an IPHS; i.e, without destroying the structure of IPHS, and therefore it can be interconnected with others IPHS and interpreted within the framework of the energy-Casimir plus damping design for control purposes. Next, we calculate the time derivative of the entropy of the closed-loop system

$$\begin{aligned}\frac{dS}{dt} &= \frac{\partial S^\top}{\partial x}J_0\frac{\partial U_{cl}}{\partial x} + R\frac{\partial S^\top}{\partial x}J\frac{\partial U_{cl}}{\partial x}\\ &= \underbrace{R\frac{\partial S^\top}{\partial x}J\frac{\partial U}{\partial x}}_{\sigma(t)} + \underbrace{R\frac{\partial S^\top}{\partial x}J\frac{\partial U_c}{\partial x}}_{\sigma_c(t)}\end{aligned}$$

where $\sigma \geq 0$ is the internal entropy variation of the system and σ_c is the external entropy variation due to the inputs of the system. As the control objective is to set a desired entropy, $\sigma_c = -\sigma$ in steady state.

The time variation of the closed-loop energy function is now given by

$$\frac{dU_d}{dt} = y_d^\top u_i \tag{3.2.11}$$

Since the internal energy function of irreversible thermodynamic systems does not have a strict minimum, it does not qualify as a candidate Lyapunov function. A standard candidate Lyapunov function for control purposes is the availability function ([1], [39], [19]). The availability function (figure 3.3) uses the convexity of the internal energy with the assumption that one of the extensive variables is fixed, to construct a strictly convex extension

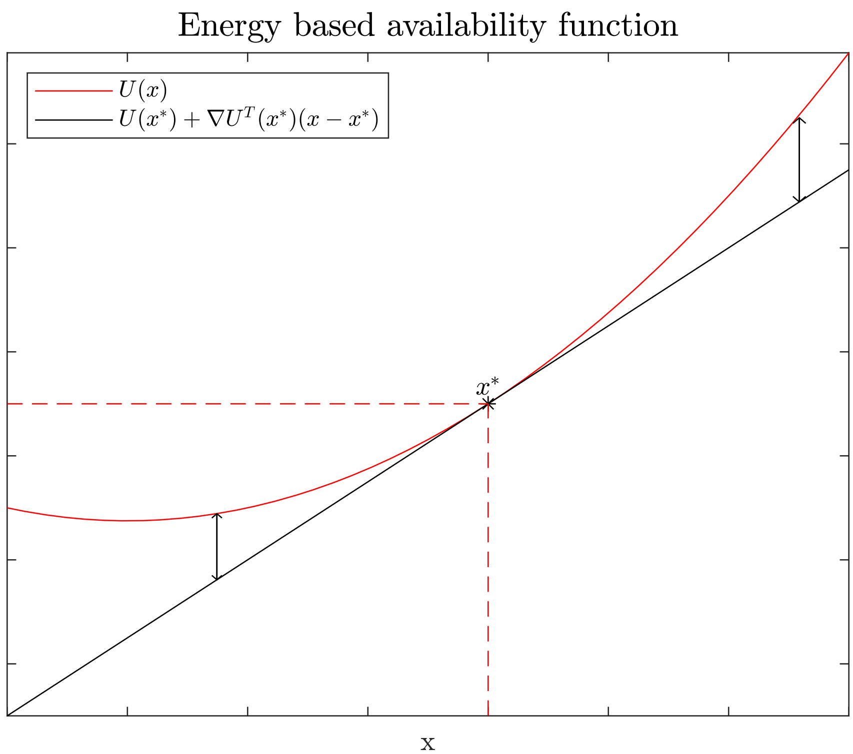

Figure 3.3. The red plot shows the internal energy of an irreversible process and the black line is the supporting hyper plane at a point x^*. The availability function is the difference between the two.

which serves as Lyapunov function for a desired dynamical equilibrium. This approach has been widely used in the control of thermodynamic systems in the last decade ([16], [17], [31]). We shall use the availability function as the target Lyapunov candidate function of the closed-loop system for the *irreversible part* of the IPHS in the energy-shaping design. The availability function is then defined as follows.

Definition 8 ([31])**.** *The energy based availability function is x defined as*

$$A(x, x^*) = U(x) - U(x^*) - \frac{\partial U}{\partial x}(x^*)^T(x - x^*) \tag{3.2.12}$$

with $U(x)$ being the internal thermodynamic energy of the system and x^* the desired equilibrium point of a thermodynamic variable x.

3.2.2 Damping Injection

The energy shaping input shapes the energy of the system into a new equilibrium dynamics and guarantees the closed-loop stability of the system in the sense of Lyapunov, but one have yet to guarantee the asymptotic stability at the equilibrium point.

Let's suppose that the closed-loop IPHS (3.2.10) has a minimum at x^* and set a damping injection input as

$$u_i = -Ky_d = -Kg^\top \frac{\partial U_d}{\partial x} \tag{3.2.13}$$

with $K = K^\top > 0$. We shall show that this control action renders the system dissipative, i.e, adds a positive definite matrix M to the structure matrix J_{ir} of the system. In fact, by setting u_i as the damping input in the closed-loop system (3.2.10) we get

$$\dot{x}(t) = (J_{ir} - gKg^T)\frac{\partial U_d}{\partial x} \tag{3.2.14}$$

The time derivative of the closed-loop energy function is then given by

$$\begin{aligned} \frac{dU_d}{dt} &= \frac{dU_d^T}{dx}(J_{ir} - gKg^T)\frac{\partial U_d}{\partial x} \\ &= \{U_d, U_d\}_{J_{ir}} - \{U_d, U_d\}_M \\ &= -\{U_d, U_d\}_M < 0 \end{aligned}$$

since $\{U_d, U_d\}_{J_{ir}} = 0$ and where $M = gKg^T \geq 0$. By Lasalle's invariance principle (Theorem 2) the closed-loop system converges asymptotically to x^*, ξ^* in the largest positively invariant set $\Omega = \{(x, \xi) \in \Re^{x \times \xi} \mid C(x, \xi) = -\kappa\}$.

Next, we give a proposition that encompasses the control by interconnection plus damping injection for IPHS.

Proposition 3. *Let Σ be an IPHS system of order n given by Definition 5 and Σ_c be an IPHS controller of order n_c given in (3.2.1). Consider the interconnection between Σ and Σ_c via the state modulated relation in (3.2.2). Without loss of generality, consider the positively invariant manifold $\Omega = \{(x, \xi) \in \Re^{x \times \xi} \mid C(x, \xi) = -\kappa\}$ such that for every relation $\xi_i - F_i(x) = \kappa_i, i = 1, ..., l \leq n_c \leq n$ it satisfies the PDE (3.2.8) and where $F = [F_1, ..., F_l] \in \Re^l, \kappa = [\kappa_1, ..., \kappa_l] \in \Re^l$ are a collection of smooth functions of the state space x, and a collection of constants that depend on the initial states of the system and the controller, respectively. The collection $C(x, \xi)$ are Casimirs of the closed-loop system if the collection of smooth functions F satisfy the PDE*

$$\begin{aligned} \frac{\partial F^T}{\partial x}(x) J_{ir} &= g_c \beta g^T \\ \frac{\partial F^T}{\partial x} J_{ir} \frac{\partial F}{\partial x} &= \bar{R} J_c \end{aligned}$$

If there exists a function $U_c(\xi)$ such that the closed-loop Hamiltonian energy function $U_d(x)$ is a candidate for a Lyapunov function and has a strict minimum at the point (x^*) for a particular election of ξ^*, then (x^*, ξ^*) is a stable equilibrium point for the closed-loop system. Furthermore, the control action

$$u_i = -Ky_d = -Kg^\top \frac{\partial U_d}{\partial x}$$

for a certain $K = K^\top \geq 0$ such that $M = gKg^\top \geq 0$ renders the system asymptotically stable.

Proof. The proof has been shown in subsections (3.2.1) and (3.2.2) □

3.2.3 Cbi-Di control of the CSTR system

In this section a Cbi-Di controller for the CSTR system is obtained. Proposition 3 shall be used in order to design the controller.

The CSTR system has states $x = \begin{bmatrix} n_1 & n_2 & \cdots & n_m & S \end{bmatrix}^\top$. We shall parametrize the design and look for Casimir functions of the form $C_1(n_1, \xi_1) = F_1(n_1) - \xi_1, \cdots, C_m(n_m, \xi_m) = F_m(n_m) - \xi_m$ and $C_{m+1}(S, \xi_{m+1}) = F_{m+1}(S) - \xi_{m+1}$ such that $\xi_i = F_i(n_i) + \kappa_i$ with $i = 1, ..., m$ and $\xi_{m+1} = F_{m+1}(S) + \kappa_{m+1}$.

We take a purely irreversible IPHS controller (2.2.1) as the CSTR is also purely irreversible; the controller then takes the form

$$\begin{aligned} \dot{\xi} &= R_c J_c \frac{\partial U_c}{\partial \xi} + g_c u_c \\ y_c &= g_c^\top \frac{\partial U_c}{\partial \xi} \end{aligned} \tag{3.2.15}$$

where $\xi = \begin{bmatrix} \xi_1 & \cdots & \xi_{m+1} \end{bmatrix}^\top \in \Re^{m+1}$, $J_c \in \Re^{m+1 \times m+1}$, $\beta \in \Re$ a scalar function and the input map

$$g_c = \begin{bmatrix} g_{11} & g_{12} \\ \vdots & \vdots \\ g_{(m+1)1} & g_{(m+1)2} \end{bmatrix}$$

where each term $g_{ij} = g_{ij}(\xi)$ can be dependent on the states of the system.

Since the CSTR is purely irreversible, we set as desired Hamiltonian energy function for the closed-loop system, the energy based availability function

$$A(t) = U_d = U(x) - [U(x^*) + \frac{\partial U^T}{\partial x}(x^*)(x - x^*)] \tag{3.2.16}$$

where x^* is the new equilibrium point. A simple choice for the energy of the controller is to take $U_c = U_d - U$, hence

$$\begin{aligned} U_c &= -[U(x^*) + \frac{\partial U^T}{\partial x}(x^*)(x - x^*)] \\ &= \sum_{i=1}^{m} (-\mu_i^* n_i + \mu_i^* n_i^*) + (-T^* S + T^* S^*) - U(n_1^*, \cdots, n_m^*, S^*) \end{aligned}$$

The parametrization of the Casimir function and the election of the Controller leads to the following condition on F

$$\frac{\partial F}{\partial x} = \begin{bmatrix} -\mu_1^* & 0 & \cdots & 0 \\ 0 & \ddots & \cdots & 0 \\ \vdots & \cdots & -\mu_m^* & \vdots \\ 0 & 0 & \cdots & -T^* \end{bmatrix}$$

where by integration we have $F(n_i) = -n_i \mu_i^*,\ i = 1, ..., m$ and $F(S) = -T^* S$. Notice that the CSTR system is purely irreversible with structure matrix $J_{ir} = RJ$ and $J_0 = 0$. By applying the first of the matching equations (3.2.8) it follows

$$\frac{\partial F^\top}{\partial x} J_{ir} = R \begin{bmatrix} 0 & \cdots & 0 & -\mu_1^* \bar{\nu}_1 \\ 0 & \cdots & 0 & \vdots \\ \vdots & \ddots & \vdots & -\mu_m^* \bar{\nu}_m \\ T^* \bar{\nu}_1 & \cdots & T^* \bar{\nu}_m & 0 \end{bmatrix}$$

And the right side of the first equation is

$$\beta g_c g^\top = \beta \begin{bmatrix} g_{11}\bar{n}_1 & \cdots & g_{11}\bar{n}_m & g_{11}\phi + \frac{g_{12}}{T} \\ g_{21}\bar{n}_1 & \cdots & g_{21}\bar{n}_m & g_{21}\phi + \frac{g_{22}}{T} \\ \vdots & \ddots & \vdots & \vdots \\ g_{(m+1)1}\bar{n}_1 & \cdots & g_{(m+1)1}\bar{n}_m & g_{(m+1)1}\phi + \frac{g_{(m+1)2}}{T} \end{bmatrix}$$

By equality, $g_{i1} = 0$ and $g_{i2} = -T\mu_i^* \bar{\nu}_i, \forall i = 1, ..., m$ with

$$\beta = R \qquad g_{(m+1)2} = -T g_{(m+1)1}\phi \qquad g_{(m+1)1} = T^* \frac{\bar{\nu}_i}{\bar{n}_i}$$

The equality $g_{(m+1)1}\frac{\bar{n}_i}{\bar{\nu}_i} = T^*$ has to be true $\forall i = 1, ..., m$. The system has a solution if the relation

$$\frac{\bar{n}_1}{\bar{\nu}_1} = \cdots = \frac{\bar{n}_m}{\bar{\nu}_m} \tag{3.2.17}$$

holds for every $i = 1, ..., m$. In [30], for batch reactors the equality (3.2.17) is the expression of De Donder's extent of reaction

$$\frac{n_{0i} - n_i}{\bar{\nu}_i} = \delta$$

where this property can be extended to the CSTR under Assumption 1. This condition has been obtained in [31] where an IDA-PBC like approach is used to design a controller for a class of CSTR. Then the input map g_c of the controller, can be written as

$$g_c = \begin{bmatrix} 0 & -T\mu_1^* \bar{\nu}_1 \\ \vdots & \vdots \\ 0 & -T\mu_m^* \bar{\nu}_m \\ T^*/\delta & TT^*\phi(\xi)/\delta \end{bmatrix} \tag{3.2.18}$$

The structure matrix J_c and the modulating function R_c of the controller are defined by the third equation in (3.2.8)

$$\frac{\partial F^T}{\partial x} J_{ir} \frac{\partial F}{\partial x} = \frac{rV}{T} T^* \begin{bmatrix} 0 & \cdots & 0 & \mu_1^* \bar{\nu}_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & & & \mu_m^* \bar{\nu}_m \\ -\mu_1^* \bar{\nu}_1 & \cdots & -\mu_m^* \bar{\nu}_m & 0 \end{bmatrix}$$

As the right hand side of the third matching equation (3.2.8) is $R_c J_c$, then it follows that

$$J_c = \begin{bmatrix} 0 & \cdots & 0 & \mu_1^* \bar{\nu}_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & & & \mu_m^* \bar{\nu}_m \\ -\mu_1^* \bar{\nu}_1 & \cdots & -\mu_m^* \bar{\nu}_m & 0 \end{bmatrix} \qquad R_c = \frac{rV}{T} T^* \tag{3.2.19}$$

Note that by the parametrization of the Casimir functions, i.e, $\xi_i = F_i(n_i) + \kappa_i,\ i = 1, ..., m$ and $\xi_{m+1} = F_{m+1}(S) + \kappa_{m+1}$, then the controller energy can be written as

$$U_c(\xi) = \sum_{i=1}^{m}(F_i(n_i) + \kappa_i) + (F_{m+1}(S) + \kappa_{m+1}) - U(n_1^*, ..., n_m^*, S^*) \tag{3.2.20}$$

Hence, $\frac{\partial U_c^T}{\partial \xi} = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}$. The IPHS controller then takes the form

$$\begin{bmatrix} \dot{\xi}_1 \\ \vdots \\ \dot{\xi}_{m+1} \end{bmatrix} = \frac{rV}{T} T^* \begin{bmatrix} 0 & \cdots & 0 & \mu_1^* \bar{\nu}_1 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & & & \mu_m^* \bar{\nu}_m \\ -\mu_1^* \bar{\nu}_1 & \cdots & -\mu_m^* \bar{\nu}_m & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & -T\mu_1^* \bar{\nu}_1 \\ \vdots & \vdots \\ 0 & -T\mu_m^* \bar{\nu}_m \\ T^*/\delta & TT^* \phi(\xi)/\delta \end{bmatrix} u_c$$

$$y_c = g_c^T \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

The energy shaping control action is then given by $u_e = -\beta g_c^T \frac{\partial U_c}{\partial \xi}$, which results

$$u_e = -\frac{rV}{T} \begin{bmatrix} T/\delta \\ -T\sum_{i=1}^{m} \mu_i^* \bar{\nu}_i - TT^* \phi(x)/\delta \end{bmatrix} \tag{3.2.21}$$

The closed-loop system then can be expressed as

$$\dot{x}(t) = \frac{rV}{T} \begin{bmatrix} 0 & \cdots & 0 & \bar{\nu}_1 \\ 0 & \cdots & 0 & \vdots \\ 0 & \cdots & 0 & \bar{\nu}_m \\ -\bar{\nu}_1 & \cdots & -\bar{\nu}_m & 0 \end{bmatrix} \begin{bmatrix} \mu_1 - \mu_1^* \\ \vdots \\ \mu_m - \mu_m^* \\ T - T^* \end{bmatrix} + \begin{bmatrix} \bar{\mathbf{n}} & \mathbf{0} \\ \phi(x) & 1/T \end{bmatrix} u_i$$

By proposition 3, a damping injection input is needed to guarantee asymptotic stability. We design $K \in \Re^{2\times 2}$ such that $M = gKg^\top \geq 0$. An easy choice is to take

$$K = \alpha \begin{bmatrix} 0 & 0 \\ 0 & T^2 \end{bmatrix}$$

for some tuning parameter $\alpha \geq 0$ which gives

$$M = \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \alpha \end{bmatrix} \in \Re^{m+1 \times m+1}$$

The gradient of the Hamiltonian energy function U_d is

$$\frac{\partial U_d}{\partial x} = \begin{bmatrix} \mu_1 - \mu_1^* & \cdots & \mu_m - \mu_m^* & T - T^* \end{bmatrix}^\top$$

and then the damping injection input, which we recall can be computed as $u_i = -Kg^T \frac{\partial U_d}{\partial x}$, takes the form

$$u_i = -\alpha \begin{bmatrix} 0 \\ T(T - T^*) \end{bmatrix} \tag{3.2.22}$$

The closed-loop system becomes

$$\begin{aligned}\dot{x} &= (-gKg^T + RJ)\frac{\partial U_d}{\partial x} \\ &= \left(\frac{rV}{T}\begin{bmatrix} 0 & \cdots & 0 & \bar{\nu}_1 \\ 0 & \cdots & 0 & \vdots \\ 0 & \cdots & 0 & \bar{\nu}_m \\ -\bar{\nu}_1 & \cdots & -\bar{\nu}_m & 0 \end{bmatrix} - \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & \cdots & \alpha \end{bmatrix}\right)\begin{bmatrix} \mu_1 - \mu_1^* \\ \vdots \\ \mu_m - \mu_m^* \\ T - T^* \end{bmatrix}\end{aligned}$$

Let us verify the time derivative of the closed-loop Hamiltonian function.

$$\frac{dU_d}{dt} = -\frac{\partial U_d^\top}{\partial x} M \frac{\partial U_d}{\partial x} = -\alpha(T - T^*)^2 \leq 0$$

where the asymptotic stability follows by Lasalle's invariance principle in a sufficient small region of $T = T^*$ under Assumption 1, which states that there is only one equilibrium for each temperature, and then $\dot{V} = 0$ only at $T = T^*$. We point out that the controller synthesized with the Cbi-Di framework for IPHS is equivalent to the one in [31] where an IDA-PBC like approach was used.

Chapter 4

A CASE STUDY: THE GAS-PISTON SYSTEM

This chapter presents the classical gas-piston system as a case study to illustrate the results of this thesis. The gas-piston system expresses reversible-irreversible phenomenon and therefore can be expressed as a coupled PHS-IPHS, and it also represents a complex mechanic-thermodynamic system. Simulations which evaluate the performance of the Cbi-Di controller are given at the end of the section.

4.1 Coupled PHS-IPHS model of the Gas-Piston system

The system analyzed is shown in Figure 4.1. A perfect gas is contained in a cylinder enclosed by a moving piston with no exchange of matter and a spring is attached to the moving piston. We assume that the piston is not submitted to gravity. Since the system has a reversible-irreversible phenomena, its behavior can be splitted in two energy analysis.

Firstly, the mechanical energy can be expressed as $H(q,p) = \frac{1}{2m}p^2 + \frac{1}{2}Kq^2$ where p is the kinetic momentum, m the mass of the piston, K is the Hooke's constant of the spring and q the relative position of the piston.

The perfect gas can be defined by its internal energy $U_{gas}(S,V)$ which is a function of the entropy S of the system and the volume V of the perfect gas. The total energy of the

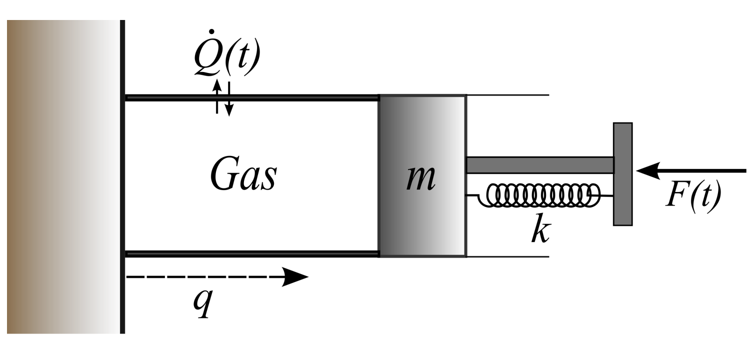

Figure 4.1. Gas piston system: a perfect gas contained in a cylinder closed by a moving piston with no exchange of matter.

system can then be expressed as the sum of both energies as

$$U(S,V,q,p) = H + U_{gas} = \frac{1}{2m}p^2 + \frac{1}{2}Kq^2 + U_{gas}(S,V) \tag{4.1.1}$$

where $x = [S,V,q,p]^\top$ is the state space vector of the system. The gradient of the total energy of the system is defined by

$$\nabla U(S,V,q,p) = \begin{bmatrix} T & -P & Kq & v \end{bmatrix}^\top$$

where T is the temperature, P the pressure and v the velocity of the moving piston. We suppose that the gas in the cylinder is subject to a non-reversible transformation due to the mechanical friction when the piston moves; we assume a non-adiabatic transformation and that the dissipated mechanical energy is transformed completely into a heat flow in the gas. The opposing force due to friction can then be expressed as $F_r = \nu v, \nu > 0$ and the force of the spring is $F_s = Kq$ due to Hooke's law. The entropy balance equation takes the form

$$\frac{dS}{dt} = \frac{1}{T}\nu v^2$$

which represents the irreversible entropy flow at temperature T induced by the heat flow νv^2 due to the friction of the moving piston. The volume of the system can be written in terms of the area and length of the moving piston as $V(t) = Aq(t)$.

The gas-piston system is submitted to an external force u_2 which is acting on the piston and symbolized as $F(t)$ in Figure 4.1. There exists an exchange of heat between the walls of the piston and the exterior, which is described by the input u_1 and described by $\dot{Q}(t)$.

The set of equations that describe the IPHS is the following

$$\begin{aligned}
\frac{dS}{dt} &= \frac{1}{T}\nu v^2 + u_1 \\
\frac{dV}{dt} &= A\frac{dq}{dt} = Av \\
\frac{dq}{dt} &= v \\
\frac{dp}{dt} &= AP - F_r - F_s + u_2 = AP - \nu v - Kq + u_2
\end{aligned}$$

where AP is the force that acts on the piston. The IPHS can be expressed as

$$\begin{bmatrix} \dot{S} \\ \dot{V} \\ \dot{q} \\ \dot{p} \end{bmatrix} = \left(R \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}}_{J} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A \\ 0 & 0 & 0 & 1 \\ 0 & -A & -1 & 0 \end{bmatrix}}_{J_0} \right) \underbrace{\begin{bmatrix} T \\ -P \\ Kq \\ v \end{bmatrix}}_{\nabla U} + \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}}_{g=[g_1,g_2]} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u} \tag{4.1.2}$$

The system can be expressed following Definition 5 as a coupled PHS-IPHS with modulating function

$$R = \frac{\nu v}{T} \tag{4.1.3}$$

and interconnection structure matrix $J_{ir} = J_0 + RJ$, where it is clear that it satisfies the skew-symmetric property $J_{ir} = -J_{ir}^T$. The temperature of the system is modelled as an exponential function of the entropy $T(S) = T_0 e^{S/c}$ ([7]) where T_0 and c are constants that depend on the system. Finally, the temperature, the volume and the pressure of the gas inside the piston can be related with the law of the ideal gases as $PV = rTN$, where N is the number of moles and r the ideal gas constant.

4.1.1 Cbi-Di control of the Gas-Piston system

In this section, we apply Proposition 3 to synthesize a Cbi-Di controller for the gas-piston system. This system expresses irreversible phenomena in the entropy S and volume V of the system; the momentum p and the position q express the reversible part of the system. Note that the position and the volume are correlated by $V(t) = Aq(t)$. Then, if a certain equilibrium q^* is imposed, then a certain equilibrium $V^* = Aq^*$ is obtained.

The design shall be divided in two parts: in a first approach, we design a controller which stabilizes the system at the point $(S, V, q, p) = (S, V^*, q^*, 0)$ by using the input u_2, which is the force acting on the piston; in a second approach, a controller is designed to control the purely irreversible process S of the system, by using the input u_1 which is the exchange of heat.

Step 1: Control of q,p and V

We look for Casimir functions for the system (4.1.2) of the form $C(x, \xi) = F(x) - \xi$ such that $\xi - F(x) = \kappa$. The aim is to render the mechanical part of the system, which relates the position q and the momentum p of the system with the input u_2. We take then as our input map

$$g_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^\top$$

The IPHS controller is defined then by $x_c = \xi \in \Re$, $J_c \in \Re$, $\beta \in \Re$ a scalar, and the input map $g_c \in \Re$, where we take for simplicity $g_c = 1$. The controller then takes the form

$$\dot{\xi} = R_c J_c \frac{\partial U_c}{\partial \xi} + u_c \tag{4.1.4}$$

where U_c is the energy of the controller. By applying the matching equations (3.2.8), the first one gives

$$\begin{aligned} \frac{\partial F^\top}{\partial x} J_{ir} &= \begin{bmatrix} -R\frac{\partial F}{\partial p} & -A\frac{\partial F}{\partial p} & -\frac{\partial F}{\partial p} & R\frac{\partial F}{\partial S} + A\frac{\partial F}{\partial V} + \frac{\partial F}{\partial q} \end{bmatrix} \\ g_c \beta g_1^\top &= \begin{bmatrix} 0 & 0 & 0 & \beta \end{bmatrix} \end{aligned}$$

From the equality it follows that

$$\frac{\partial F}{\partial p} = 0, \quad R\frac{\partial F}{\partial S} + A\frac{\partial F}{\partial V} + \frac{\partial F}{\partial q} = \beta$$

The election and solution of the PDE is motivated by the election of a proper energy function for the system. Since the system has a reversible-irreversible phenomenon, we use the framework of the availability function for the irreversible part. Further, the aim is to render the closed-loop Hamiltonian function as

$$U_{d1}(S, V, q, p) = A_1(S, V) + \frac{1}{2m} p^2 + \frac{1}{2}(K + K_0)(q - q^*)^2$$

where $A_1(S,V) = U_{gas}(S,V) - [U_{gas}(S^*,V^*) - P^*(V - V^*)]$ is the availability function (8) for the irreversible coordinate, associated to the volume of the system. We recall from (4.1.1) that the internal energy of the system is

$$U(S,V,q,p) = H + U_{gas} = \frac{1}{2m}p^2 + \frac{1}{2}Kq^2 + U_{gas}(S,V)$$

Since the energy of the closed-loop system can also be expressed as $U_{d1} = H + U_{gas} + U_c$, then the simplest choice for the election of U_c is given by

$$\begin{aligned} U_c &= U_{d1} - U \\ &= P^*(V - V^*) - U_{gas}(S^*,V^*) + \frac{1}{2}K_0q^2 - (K+K_0)qq^* + \frac{1}{2}(K+K_0)(q^*)^2 \end{aligned}$$

The controller energy suggests that the following elections

$$\frac{\partial F}{\partial p} = 0, \quad \frac{\partial F}{\partial S} = 0, \quad \frac{\partial F}{\partial V} = \alpha_1, \quad \frac{\partial F}{\partial q} = \alpha_2 + \alpha_3 q$$

which gives $F = \alpha_1 V + \alpha_2 q + \frac{\alpha_3}{2}q^2$, allow to express the energy of the controller as $U_c = F + \kappa = \xi$ with $\alpha_1 = -P^*$, $\alpha_2 = -(K+K_0)q^*$, $\alpha_3 = K_0$ and $\kappa = -P^*V^* - U_{gas}(S^*,V^*) + \frac{1}{2}(K+K_0)q_0^2$, which results in $\frac{\partial U_c}{\partial \xi} = 1$.

The third equation of (3.2.8) gives the condition on the structure matrix J_c of the controller, which results in $J_c = 0$ and $\beta = AP^* - (K+K_0)q^* + K_0q$. The IPHS controller can then be written as the simple integrator

$$\dot{\xi} = u_c$$

The energy shaping control action is given by

$$u_{e1} = -\beta g_c^\top \frac{\partial U_c}{\partial \xi} = -AP^* + (K+K_0)q^* - K_0q \tag{4.1.5}$$

By Proposition 3, for the damping injection, we take $K_1 \in \Re$ such that $M_1 = g_1K_1g_1^\top \in \Re^{4\times4} \geq 0$. An easy choice is to take $K_1 = \alpha_1 \geq 0$ with α_1 a tuning parameter, which gives

$$M_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_1 \end{bmatrix}$$

a semi positive define matrix. The damping injection control input becomes

$$u_{i1} = -K_1g_1^\top \frac{\partial U_{d1}}{\partial x} = -\alpha_1\frac{p}{m} = -\alpha_1 v \tag{4.1.6}$$

The final control input is then given by $u_1 = u_{e1} + u_{i1}$.

The input u_{e1} shapes the energy of the system (4.1.2), which now has a strict minimum at the point $(S,V,q,p) = (S,V^*,q^*,0)$. Notice that the gas-piston system (4.1.2) can be rewritten as

$$\begin{bmatrix} \dot{S} \\ \dot{V} \\ \dot{q} \\ \dot{p} \end{bmatrix} = \left(R \underbrace{\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}}_{J} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A \\ 0 & 0 & 0 & 1 \\ 0 & -A & -1 & 0 \end{bmatrix}}_{J_0} \right) \underbrace{\begin{bmatrix} T \\ -(P-P^*) \\ (K+K_0)(q-q^*) \\ v \end{bmatrix}}_{\nabla U_{d1}} + \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}}_{g=[g_1, g_2]} \underbrace{\begin{bmatrix} 0 \\ u_2 \end{bmatrix}}_{u} \tag{4.1.7}$$

Step 2: Control of S

Next, in a second control step, we aim at designing a control action for the entropy of the system (4.1.7), which is the system that the input u_1 shaped. As the aim is to shape the entropy of the system at a point $S = S^*$, the desired Hamiltonian energy function, which we set as U_d, is

$$\begin{aligned} U_d(S,V,q,p) &= A_2(S,V) + U_{d1}(S,V,q,p) \\ &= (U_{gas} - U^*_{gas}) - T^*(S-S^*) + P^*(V-V^*) + \frac{1}{2m}p^2 + \frac{1}{2}(K+K_0)(q-q^*)^2 \end{aligned} \tag{4.1.8}$$

with $A_2(S,V) = -T^*(S-S^*)$ being the availability function (Definition 8) for the irreversible coordinate associated to the entropy of the system.

Let us take the time derivative of the Hamiltonian energy function (4.1.8)

$$\begin{aligned} \frac{dU_d}{dt} &= \frac{dU^\top}{dx}\frac{dx}{dt} \\ &= \nabla U_d^\top (J_{ir}\nabla U_{d1} + gu) \\ &= \nabla U_d^\top J_{ir}\nabla U_{d1} + \nabla U_d^\top g_2 u_2 \end{aligned} \tag{4.1.9}$$

We look for an input $u_2 = u_{e2}$ such that the time derivative of the desired Hamiltonian energy $U_d(t)$ of the system (4.1.7) satisfies

$$\frac{dU_d}{dt} = \nabla U_d^\top J_{ir}\nabla U_{d1} + \nabla U_d^\top g_2 u_{e2} \leq 0 \tag{4.1.10}$$

Where the gradient of U_d is

$$\nabla U_d = \begin{bmatrix} T-T^* & -(P-P^*) & (K+K_0)(q-q^*) & p/m \end{bmatrix}^\top$$

Expanding (4.1.10) and eliminating terms, it follows

$$\begin{aligned} \frac{dU_d}{dt} &= \nu\left(\frac{p}{m}\right)^2 \frac{(T-T^*)}{T} - \nu\left(\frac{p}{m}\right)^2 + (T-T^*)u_{e2} \\ &= (T-T^*)\left(u_{e2} + \nu\left(\frac{p}{m}\right)^2\frac{1}{T}\right) - \nu\left(\frac{p}{m}\right)^2 \end{aligned}$$

If we select the input to be

$$u_{e2} = -\nu\left(\frac{p}{m}\right)^2\frac{1}{T} \tag{4.1.11}$$

then it is clear that

$$\frac{dU_d}{dt} = -\nu \left(\frac{p}{m}\right)^2 \leq 0, \forall t \tag{4.1.12}$$

A damping injection input can also be added; taking $K_2 \in \Re$ such that $M_2 = g_2 K_2 g_2^\top \in \Re^{4\times 4} \geq 0$ with $K_2 = \alpha_2 \geq 0$, gives

$$M_2 = \begin{bmatrix} \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

a semi positive definite matrix. The damping injection control input is then

$$u_{i2} = -K_2 g_2^\top \frac{\partial U_d}{\partial x} = -\alpha_2 (T - T^*) \tag{4.1.13}$$

The final control input is then $u_2 = u_{e2} + u_{i2}$. The complete closed-loop system can be written as

$$\begin{bmatrix} \dot{S} \\ \dot{V} \\ \dot{q} \\ \dot{p} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} 0 & 0 & 0 & R \\ 0 & 0 & 0 & A \\ 0 & 0 & 0 & 1 \\ -R & -A & -1 & 0 \end{bmatrix}}_{J_{ir}} - \alpha_1 \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{M_1} - \alpha_2 \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{M_2} \right) \underbrace{\begin{bmatrix} T - T^* \\ -(P - P^*) \\ (K + K_0)(q - q^*) \\ v \end{bmatrix}}_{\nabla U_d} \tag{4.1.14}$$

where the control action

$$u = \underbrace{\begin{bmatrix} -\nu \left(\frac{p}{m}\right)^2 \frac{1}{T} \\ (K + K_0) q^* - K_0 q - A P^* \end{bmatrix}}_{u_e} + \underbrace{-\alpha_1 \begin{bmatrix} 0 \\ \frac{p}{m} \end{bmatrix} - \alpha_2 \begin{bmatrix} (T - T^*) \\ 0 \end{bmatrix}}_{u_i} \tag{4.1.15}$$

guarantees the asymptotic stability of the system at the desired equilibrium point.

4.2 Numerical Simulations

In this section simulations are shown for the gas-piston system. The Cbi-Di controller synthesized in section 4.1.1, equation (4.1.15) is simulated in Matlab-Simulink for different values of interest.

4.2.1 Simulated Cases

Firstly, as equation (4.1.15) shows, there are three tuning parameters: K_0 which changes the spring value and the damping parameters α_1, α_2. For simplicity, we take $\alpha_1 = \alpha_2 = \alpha$. The sum $K + K_0$, which is the total spring constant, has to be positive hence $K_0 > -K$.

As we take $\alpha_1 = \alpha_2 = \alpha$, i.e, equal damping parameters, there are two tuning parameters in K_0 and α. In a first simulation, K_0 shall remain constant while α takes different values; next, α remains constant while K_0 takes different values.

4.2.2 Simulation Values

Table 4.1 shows the parameters which describes the Gas-Piston system with its simulation values. The control objective is the volume and entropy of the piston, with a desired volume of $V^* = 0.03[m^3]$ which gives $q^* = 0.6[m]$; momentum $p^* = 0[Kg{\cdot}m/s]$ and a desired entropy of $S^* = 30[J/K]$.

Numerical values were taken from general applications that include electric generators, hydraulic pumps and air compressors [18] while thermodynamics values were taken from [8].

4.2.3 Simulation Results

This section shows the simulation results with the simulation values of section 4.2.2.

Figure 4.2 and 4.3 show the state variables of the system for different α values, and different K_0 values, respectively. The objective variables are the volume and the entropy of the system. The plot shows the entropy S of the system; the volume V of the piston and the momentum p. We didn't explicit show the position of the piston as this variable has a linear behaviour with the volume of the piston.

The K_0 parameter, which varies in figure 4.3 and which is related to the closed-loop Hamiltonian function, shows that there is a trade off between the speed of the closed-loop system and the oscillation of the states. The input u_2 in figure 4.5, when K_0 is moving, shows that as K_0 increases more energetic inputs we get; this is natural as more energetic inputs are needed in order for the closed-loop to converge faster.

The entropy of the system, otherwise, has no relation with the tuning parameter K_0 which is shown in input u_1, figure 4.4 when $\alpha = 1$. This is no surprise as the control action acting on the entropy is entirely due to the heat flux.

The damping injection coefficient α is in charge of the asymptotic stability of the system according to proposition 3; figure 4.2 shows the response of the system for different α values. An increase of α produces a faster response on the dynamics of the system; as a result, the control objective is reached faster. See, for example, the entropy, the volume and the momentum of the system for $\alpha = 10$. For this particular value, input u_1, u_2 in figures 4.4,4.5 with $K_0 = 0$, show that as expected, inputs with faster initial response are needed to guarantee a faster response of the closed-loop system.

The choice of α and K_0 depends on the control requirements. If a faster response of the closed-loop system is required, with no particular need of a low magnitude input, then a high damping value should be selected.

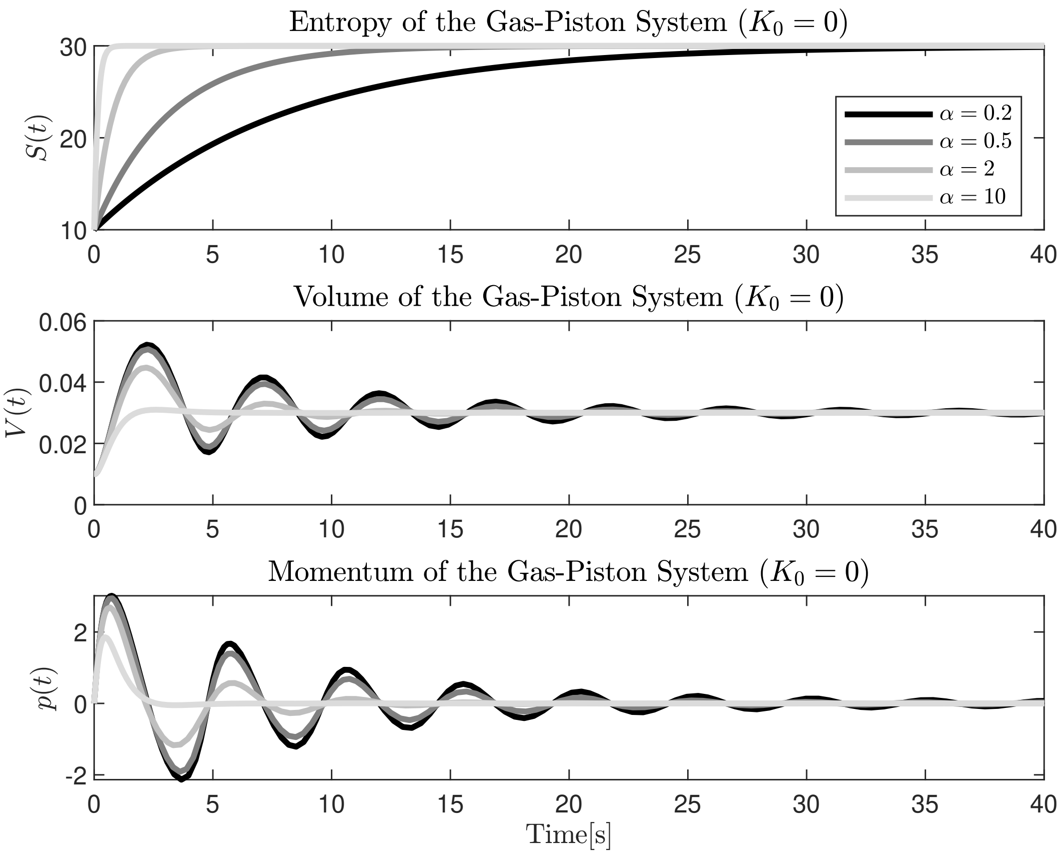

Figure 4.2. Simulation of the closed-loop system for different α values and with a fixed $K_0 = 0$.

m	Mass of the system		5 Kg		
q	Relative position of the piston	$q(0)$	0.2 m	q^*	0.6 m
p	Momentum of the system	$p(0)$	0 Kg m/s	p^*	0 Kg m/s
S	Entropy of the system	$S(0)$	10 $\mathrm{J\,K^{-1}}$	S^*	30 $\mathrm{J\,K^{-1}}$
V	Volume of the piston	$V(0)$	0.01 $\mathrm{m^3}$	V^*	0.03 $\mathrm{m^3}$
P	Presion of the system	$P(0)$	254 N/m		
ν	Friction coefficient		1 Kg/s		
v	Velocity of the system	$v(0)$	0 m/s		
A	Area of the piston		0.05 $\mathrm{m^2}$		
T	Temperature of the system	$T(0)$	306 K		
r	Ideal gas constant		8.3 $\mathrm{J\,K^{-1}\,mol^{-1}}$		
c	Constant temperature model		500 K		
N	Number of moles		0.001 mol		

Table 4.1. Variables and parameters of the gas-piston system with initial conditions.

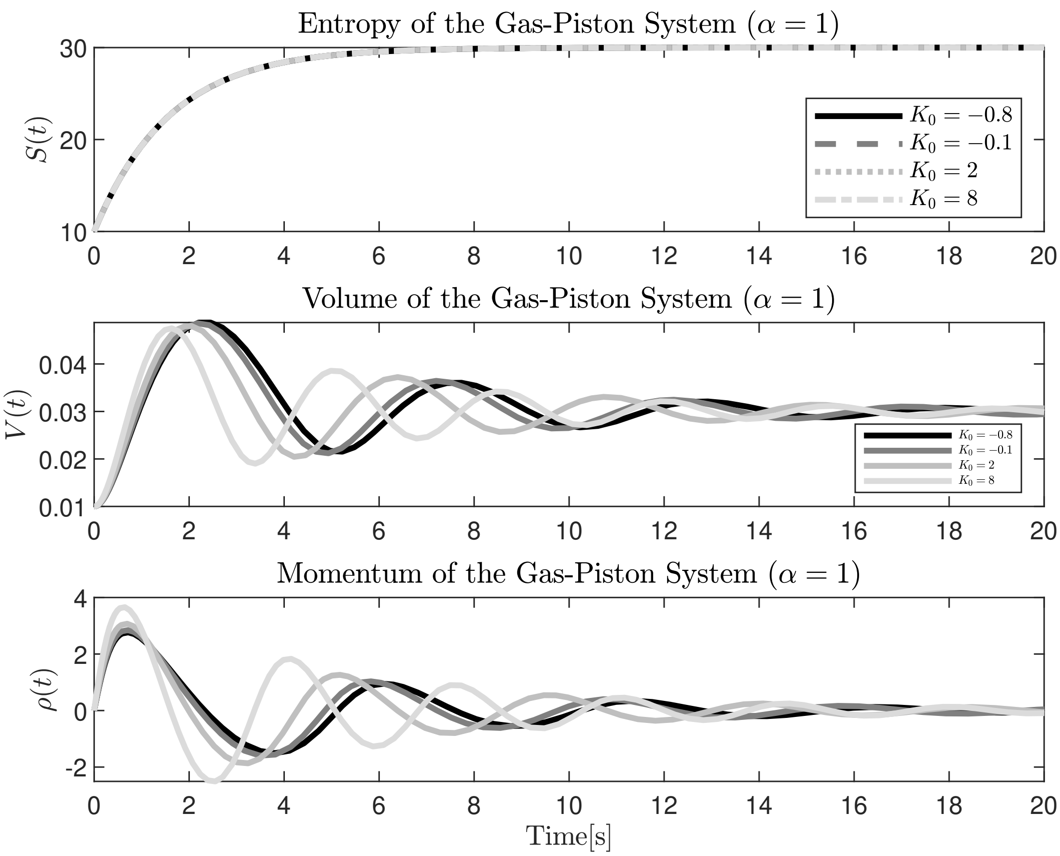

Figure 4.3. Simulation of the closed-loop system for different K_0 values and with a damping injection constant of $\alpha = 1$.

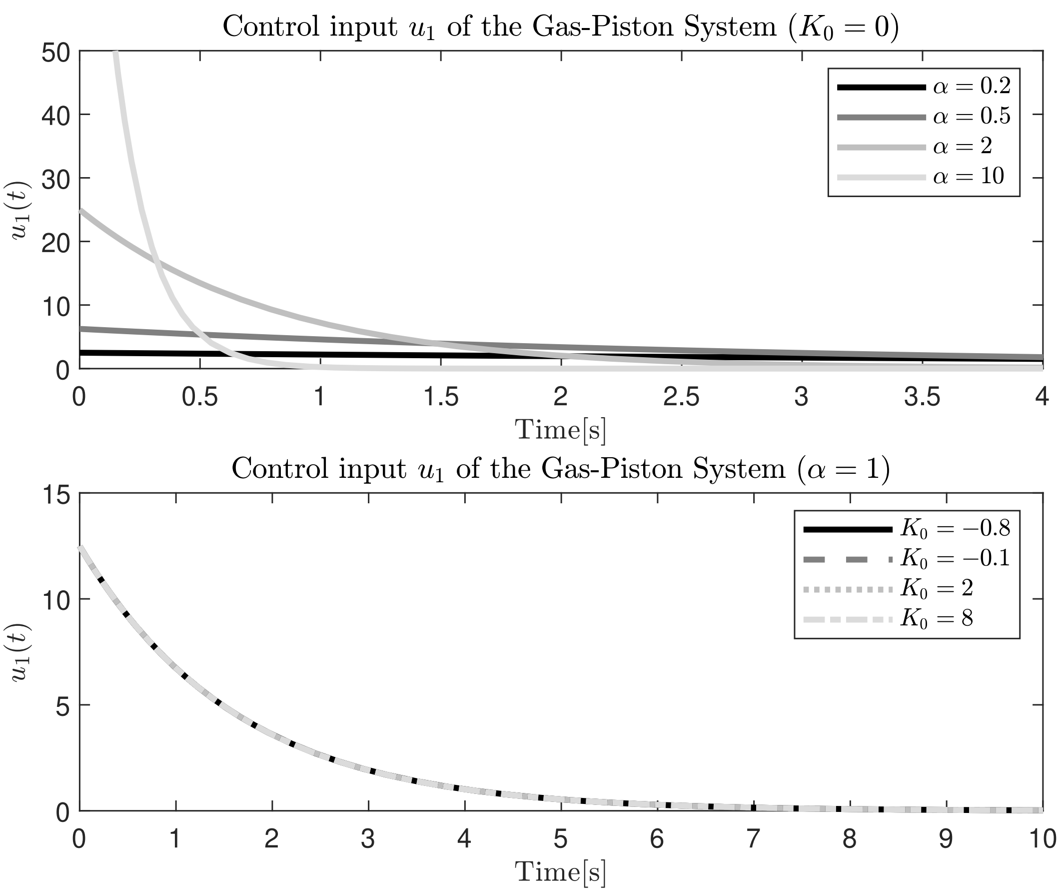

Figure 4.4. Control input action $u_1(t)$ design for the gas-piston system for different K_0, α values.

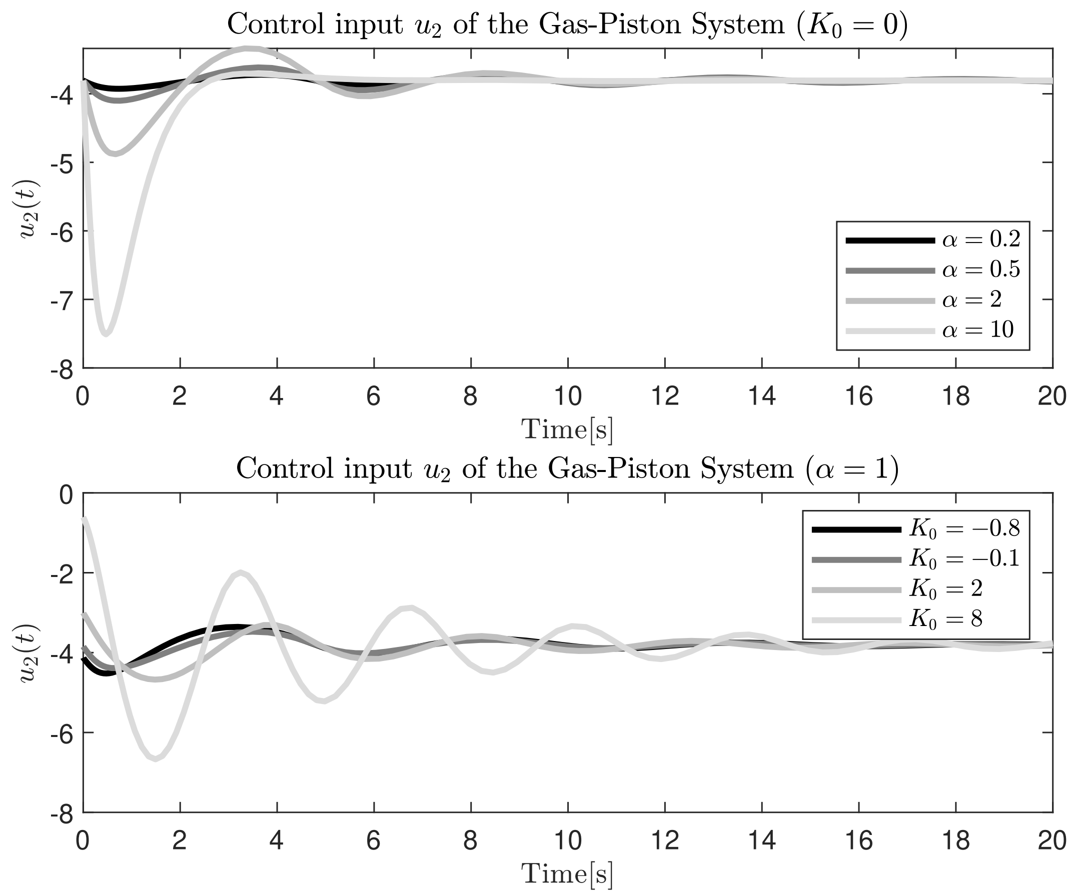

Figure 4.5. Control input action $u_2(t)$ design for the gas-piston system for different K_0, α values.

Chapter 5

CONCLUSION

This thesis considers the problem of passivity based control (PBC) of irreversible port Hamiltonian systems (IPHS) which are an extension of the port Hamiltonian system (PHS) framework for irreversible thermodynamic. The control design is based on the classical PBC techniques for PHS such as control by interconnection (Cbi) and damping injection (Di). To this purpose, an IPHS is controlled with a double control loop; the first loop is in charge of the stability of the system by shaping the Hamiltonian energy function with an energy shaping control design approach, where the Cbi is done by considering an IPHS controller. In order to achieve this, the existence of Casimir functions, which relate the states of the system and the controller, are fundamental. The result is a set of partial differential equations whose solutions are the Casimir functions of the system. The energy shaping controller then shapes the Hamiltonian energy of the closed-loop into a desired energy function.

The second control loop guarantees the asymptotic stability of the system, through a damping injection control action.

The result is then a controller which achieves asymptotic stability at a desired equilibrium dynamic in the closed-loop system.

A proposition that summarize the Cbi-Di control for IPHS is given. Furthermore, a Cbi-Di controller is obtained for a CSTR system as an illustrative example and the correspondence of the controller with previously reported results using IDA-PBC is established.

A controller for a gas-piston system, which is a complex coupled mechanic-themodynamic process, is synthesized and simulations for the system show the performance of the controller. The closed-loop system is asymptotically stable and numerical simulations illustrate how the entropy of the system, which represents the irreversibility of the process, follows the reference with a tuning parameter which determines the speed of the closed-loop system response.

5.1 Future Work

As the present thesis extends the Cbi-Di control for irreversible-reversible processes, an important application for future work is related to the control of micro-mechanical systems with hysteresis, where piezoelectric systems are of particular interest and others coupled mechanical-thermodynamical systems. Also, it would be interesting to compare the performance of the Cbi-Di controllers with other frameworks for control of irreversible-reversible systems.

Numerical simulations for the CSTR systems are also important, with the application of the method, for example, to a Van Der Vusse Reactor ([34], [24]).

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