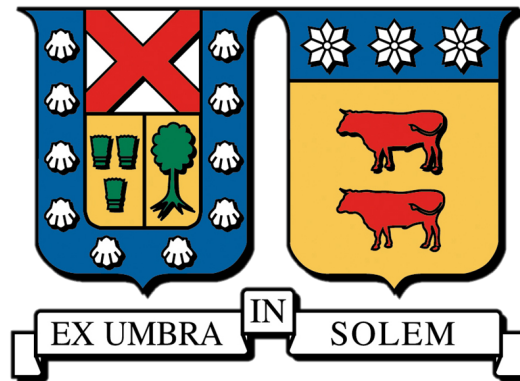


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DEPARTAMENTO DE FÍSICA



Testing the Cataclysmic Variable formation channel
for SDSS J1257+5428 with saturated Magnetic Braking

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THESIS TO OBTAIN THE DEGREE OF BACHELOR IN ASTROPHYSICS

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Abstract

Binary systems containing two white dwarfs (WDs) are objects of great interest because they are involved in extreme physics phenomena, and because they are possible progenitors of Type Ia supernovae (SN Ia). Despite significant recent progress, however, we do not properly understand how close WD binaries form and evolve. In this context, observationally well characterized systems that allow to test evolutionary scenarios are of utmost importance. Among such systems, SDSS J1257+5428 stands out, consisting of an extremely low-mass WD ($\lesssim 0.2 M_{\odot}$) with an effective temperature of $\sim 6\,400$ K, and a massive WD with a mass of $\sim 1 M_{\odot}$ with an effective temperature of $\sim 13\,000$ K. Recently, the presence of a third WD has been inferred, which provides a lower limit on the total age of the system. The properties of this system appear to be at odds with theoretical expectations, giving rise to its characterization as a paradoxical system.

We investigate whether it is possible to reproduce SDSS J1257+5428 through a previously ignored evolutionary channel, the cataclysmic variable (CV) formation channel, considering a new prescription for magnetic braking (MB). We use the MESA code to perform detailed binary evolution models and to explore whether a system evolving from a post-common-envelope configuration can reproduce the observed properties of SDSS J1257+5428. In our setup, we adopt a saturated and enhanced MB prescription.

We find that it is indeed possible to explain the existence of the paradoxical system. Our best model predicts that the companion star reached the end of the MS forming a Helium core before the onset of mass transfer. Following the mass transfer episode the companion underwent hydrogen flashes, ending as an extremely low-mass WD similar to the observed one. The WD accretor becomes more massive consistent with the observed mass.

We conclude that SDSS J1257+5428 can be explained as a descendent of a CV, using the same MB prescription used to model CVs.

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Chapter 1

Introduction

White dwarfs (WDs) represent approximately 95% of stellar remnants and correspond to the final evolutionary stage of stars with initial masses $\lesssim 8 M_{\odot}$ (Althaus et al., 2010). Their formation takes place during the late stages of the giant branch. These remnants can be found both in isolated systems and in multiple systems, particularly in binary systems, which consist of two objects orbiting a common center of mass.

Binary systems containing two WD in compact configurations, i.e., with short orbital periods, have attracted considerable interest in recent years. This is because they can act as progenitors of Type Ia supernovae (SN Ia), which are used to measure cosmological parameters (Liu et al., 2023). Moreover, due to their compact configuration, these systems are considered important sources of low-frequency gravitational waves (Korol, V. et al., 2020; Amaro-Seoane et al., 2023; Li & Chen, 2024), and serve as surviving test subjects of the common envelope phase (Nandez et al., 2015). Despite the astrophysical relevance of these objects, the mechanisms leading to the formation of double WD systems are still not fully understood.

Among double WD binaries, the system SDSS J125733.63+542850.5 (hereafter SDSS J1257+5428) stands out, having received significant attention because its evolutionary pathway cannot be easily explained within classical models of binary evolution, and it has therefore been classified as a paradoxical system. Identifying an evolutionary scenario capable of reproducing its observed properties would not only help to resolve this puzzle, but would also provide stronger constraints on binary evolution models, particularly on orbital angular momentum loss (AML) mechanisms and mass transfer processes.

In this work, we investigate one of the possible formation channels that could explain the origin of this system. This channel is based on the assumption that SDSS J1257+5428 is the descendant of a cataclysmic variable (CV), i.e., a binary system composed of an accreting WD and a main-sequence (MS) star that fills its Roche lobe. CVs are among the most extensively studied binary systems, as they constitute an ideal laboratory for exploring the physics of accretion and the evolution of close binary systems (Hameury, 2020).

To address this problem, it is first necessary to understand the different mass transfer modes that arise in close binary systems. Using these concepts, we can then examine the evolution of CVs and how it depends on the properties of the donor star. Finally, we analyze the observed properties of SDSS J1257+5428, which we aim to reproduce using numerical models, highlighting the difficulties and peculiarities that emerge when attempting to model the evolution of this system.

Chapter 2

Mass Transfer

In this chapter, the geometry of a binary system and the mechanisms that trigger mass transfer are described, together with the different regimes that may occur during mass transfer episodes. These concepts are fundamental for understanding the evolution of CVs, particularly under the assumption adopted in this work that the system studied descends from a CV and undergoes multiple mass transfer episodes throughout its evolution.

2.1 Roche Lobe geometry

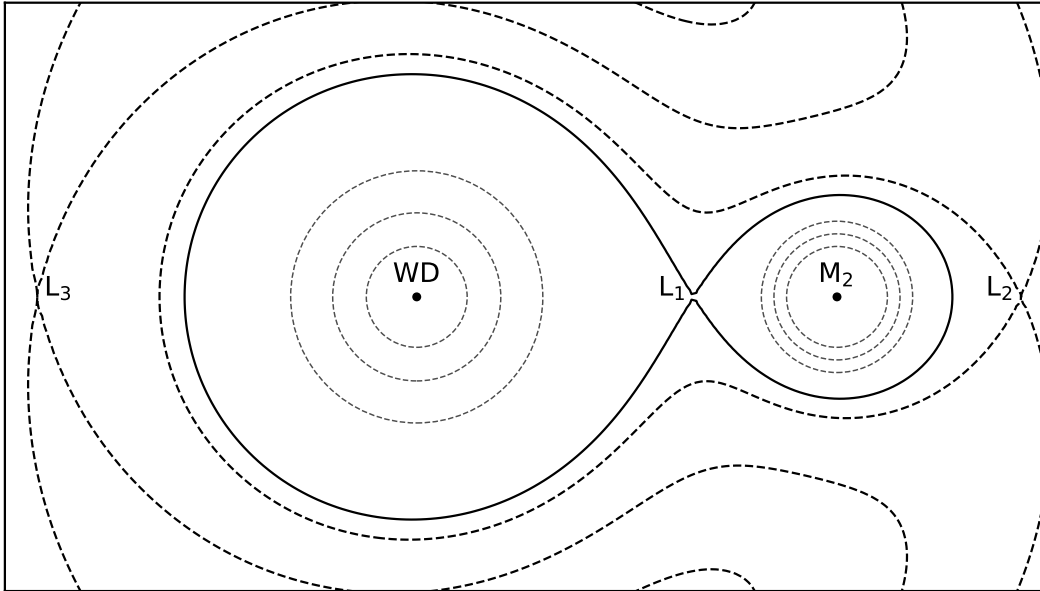


Figure 2.1: Equipotential surfaces for a binary system with a mass ratio $M_2/M_{WD} = 0.2$. The solid lines represent the Roche lobes of both components. The L_1 point corresponds to the inner Lagrange point, through which mass transfer from the donor to the WD occurs.

The gravitational and rotational potential in a corotating frame with a binary system is known as the Roche potential, and is defined as

$$\Phi_{\text{Roche}} = -\frac{GM_1}{|\vec{r} - \vec{r}_1|} - \frac{GM_2}{|\vec{r} - \vec{r}_2|} - \frac{1}{2}(\vec{\omega} \times \vec{r})^2, \quad (2.1)$$

where $\vec{r}_1$ and $\vec{r}_2$ correspond to the positions of each star, and the third term represents the centrifugal potential, with $\vec{\omega}$ is the angular velocity.

From this potential, equipotential surfaces can be defined, whose shapes allow one to identify the regions where gravitational forces dominate in the corotating frame. In Figure 2.1, the solid contours correspond to the Roche lobes of each star. These surfaces delimit the maximum region within which the material remains attached to its respective star. When a star fills its Roche lobe, mass transfer begins through the inner Lagrange point L_1 . This point is unstable along one direction, causing the material to preferentially flow toward the companion star. Roche lobe filling may occur either due to the shrinkage of the lobe as a result of orbital AML mechanisms, or due to the expansion of the stellar radius associated with stellar evolution. The stability of mass transfer is closely related to three fundamental timescales, which describe the hydrostatic and thermal response of the star, as well as the characteristic timescale associated with the evolution of its core (Tauris & van den Heuvel, 2006).

2.2 Dynamical Timescale Mass Transfer

When mass transfer occurs in a binary system, the donor star is forced out of hydrostatic equilibrium. The timescale for the star to restore this equilibrium is known as the dynamical timescale τ_{dyn} , which is much shorter than the thermal timescale τ_{KH} .

In order to restore hydrostatic equilibrium, the star must adjust its radius relative to the radius of its Roche lobe. The stability of mass transfer in this regime can be analysed by comparing the response of the donor star's radius, assuming a fixed evolutionary profile and specific entropy, with the response of the Roche lobe radius to mass loss. This criterion is defined through the mass–radius exponents (*adiabatic radius-mass exponent* and *Roche lobe radius-mass exponent*) established by Ge et al. (2010, 2015, 2020a), where the stellar response is assumed to be adiabatic. The following exponents are defined:

$$\zeta_{\text{ad}} = \left. \frac{\partial \ln R_{\text{d}}}{\partial \ln M_{\text{d}}} \right|_{\text{ad}} \quad (2.2)$$

$$\zeta_{\text{RL}} = \left. \frac{\partial \ln R_{\text{RL}}}{\partial \ln M_{\text{d}}} \right|_{\text{RL}} \quad (2.3)$$

where R_{d} and M_{d} are radius and mass of the donor star, R_{RL} is the Roche lobe radius, these exponents. Mass transfer is dynamically stable if the condition $\zeta_{\text{ad}} > \zeta_{\text{RL}}$. Otherwise if $\zeta_{\text{ad}} < \zeta_{\text{RL}}$, mass transfer

becomes dynamically unstable and proceeds on the dynamical timescale. During this process, the mass transfer rate increases rapidly, reaching very high values without the possibility of self-regulation. For MS donor stars, this scenario typically leads to the development of a common-envelope phase (more details in Ivanova et al., 2020), that can be observed at the left branch of 3.1.

2.3 Thermal Timescale Mass Transfer

When the conditions for hydrostatic equilibrium are maintained, the evolution of mass transfer can still affect the thermal equilibrium of the donor star. This equilibrium corresponds to the balance between nuclear energy production in the stellar interior and radiative energy losses through the atmosphere. Mass loss may drive the star out of this equilibrium, forcing it to readjust its structure in order to restore it on the thermal timescale.

In an analogous way to what was discussed in the previous section, the boundary between thermally stable and unstable mass transfer can be defined through the mass–radius exponent introduced by Ge et al. (2020b), which describes the response of the stellar radius once the star has regained thermal equilibrium. This *equilibrium radius–mass exponent* is defined as

$$\zeta_{\text{eq}} = \left. \frac{\partial \ln R_d}{\partial \ln M_d} \right|_{\text{th}} \quad (2.4)$$

Mass transfer is dynamically and thermally stable if its satisfied $\zeta_{\text{eq}} > \zeta_{\text{RL}}$ and $\zeta_{\text{ad}} > \zeta_{\text{RL}}$. Contrarily, if $\zeta_{\text{eq}} < \zeta_{\text{RL}}$ and $\zeta_{\text{ad}} > \zeta_{\text{RL}}$ mass transfer is dynamically stable but thermally unstable, and therefore proceeds on the thermal timescale. In this regime, the mass transfer rate increases until it reaches a saturation level, which is set by the thermal readjustment of the donor star itself, ultimately allowing the star to recover thermal equilibrium.

2.4 Nuclear or/and Angular Momentum Loss

If mass transfer is dynamically and thermally stable, its evolution can be driven by external agents associated with stellar evolution or with the evolution of the binary system itself. In these cases, mass transfer is not determined by the immediate structural response of the donor star, but rather by the processes that force the system to maintain Roche lobe contact.

On the one hand, the donor star may fill its Roche lobe as a consequence of its nuclear evolution, when at certain evolutionary stages its radius increases until it becomes comparable to the Roche lobe radius ($R_d \sim R_{\text{RL}}$). In this scenario, mass transfer proceeds on the nuclear timescale. On the other hand, the Roche lobe may shrink as a result of orbital AML from the system. In this case, even if the radius of the donor star remains approximately constant (or even shrinks), the contraction of the Roche lobe can force

the onset or continuation of mass transfer, which then occurs on the timescale associated with AML.

If the evolution of mass transfer is driven by orbital AML, its behaviour will depend directly on the strength and nature of the mechanism responsible for that loss.

2.4.1 Systematic Angular Momentum Loss

The main mechanisms of systematic angular momentum loss that drive the evolution of binary systems are gravitational wave radiation (Paczynski, 1967, GR) and magnetic braking (MB).

Using the weak-field approximation in general relativity, an expression can be derived for the rate of change of orbital angular momentum due to GR emission in a two-body system (Hurley et al., 2002, Eq. 48). This mechanism operates in all binary systems; however, due to its strong dependence on the orbital separation, AML through GR becomes particularly efficient in compact binaries.

The main physical process underlying MB is that highly ionized gas, expelled through stellar winds, is forced to corotate with the stellar magnetic field out to the Alfvén radius. During this process, the wind particles extract angular momentum from the star, causing it to gradually spin down. In the case of a binary system, if one of the stars is tidally locked to the orbital period, the loss of stellar angular momentum is transferred to the orbit, driving the system toward shorter orbital periods. Despite the basic physical mechanism of MB is relatively simple, the dependencies of its various components are not. The strength of the torques, the Alfvén radius, and the mass-loss rate through stellar winds depend on the strength and geometry of the stellar magnetic field, whose origin and evolution are still not well-understood. This has motivated several attempts to construct empirical and observationally motivated models in order to place constraints on MB. At present, there is no single model that can be applied consistently across all stellar masses or populations (Belloni & Schreiber, 2023, for a review). Nevertheless, one of the most robust observational constraints is the saturation of MB at short rotation periods, implying that the AML rate reaches a maximum value, both for single stars and for stars in binary systems.

2.4.2 Consequential Angular Momentum Loss

During mass transfer, a simple approximation is to assume that the total mass of the system is conserved, that is, that mass transfer is conservative. However, this assumption is not fully realistic when considering the various phenomena associated with mass transfer that can lead to the loss of part of the system’s mass.

When a fraction of the transferred mass is expelled from the system, mass transfer becomes non-conservative. In this case, the processes responsible for mass loss contribute additionally to the loss of orbital angular momentum, since the ejected material carries angular momentum away from the binary. Because this AML is a direct consequence of mass loss during mass transfer, these processes are collectively referred to as consequential angular momentum loss (CAML) mechanisms. Among the several mechanisms, the most important CAML mechanisms are the following. Fast and isotropic winds in the vicinity of the accreting

star, through which a fraction of the transferred mass is ejected from the system. Under the assumption of sufficiently fast winds, the expelled material is assumed not to interact significantly with the binary. This scenario includes isotropic re-emission from the vicinity of the accretor, produced by enhanced winds, jets in compact objects, or nova eruptions (King & Kolb, 1995). Circumbinary discs (Spruit & Taam, 2001), in which a fraction of the transferred mass feeds a disc that forms around the binary system and efficiently extracts orbital angular momentum and winds from the accretion disk (Cannizzo & Pudritz, 1988).

Chapter 3

Evolution of Cataclysmic Variables

Understanding the different mass transfer regimes allows us to better interpret the formation and evolution of CVs. In this chapter, we present a concise description of how CVs form and how they evolve over time. To clarify the importance of the assumptions adopted in this work, we explain the main evolutionary paths that can be followed by systems with low-mass donors, as well as by systems in which the donor star has an evolved core.

As in most compact accreting objects, CVs form from the evolution of an initially wide zero-age MS-MS binaries. The more massive star evolves more rapidly off the MS toward the giant branches, and during this phase it fills its Roche lobe, initiating a mass transfer episode. During these phases, in particular, during the asymptotic giant branch the star may form a carbon–oxygen degenerate or semi-degenerate core (Kippenhahn et al., 2013; Carroll & Ostlie, 2017).

Typically, during the mass transfer, the response of the giant star to keep hydrostatic equilibrium causes more mass transfer, consequently the mass transfer becomes dynamically unstable leading to a common envelope phase (Warner, 1995). During this phase, both stars spiral closer together due to drag forces, while the envelope gradually gains thermal energy. If the envelope reaches a sufficiently high energy, it is ejected from the system as a planetary nebula, leaving behind a detached binary composed of a degenerate component, i.e. a carbon–oxygen WD (CO WD), and an unevolved MS star. Otherwise, the envelope remains bound to the system and the two stars ultimately merge.

3.1 CV evolution with low-mass donors (M-/K-Type)

In the case of a low-mass donor star ($\lesssim 0.8, M_{\odot}$), the star on the MS has a radiative core and a convective envelope. As a consequence, MB efficiently removes orbital angular momentum, causing the Roche lobe to shrink and driving the binary toward shorter orbital periods, as shown in Fig. 3.1.

At some point, the donor fills its Roche lobe, triggering a phase of mass transfer and transforming the system into a CV. During this stage, the high efficiency of MB leads to a large mass transfer rate.

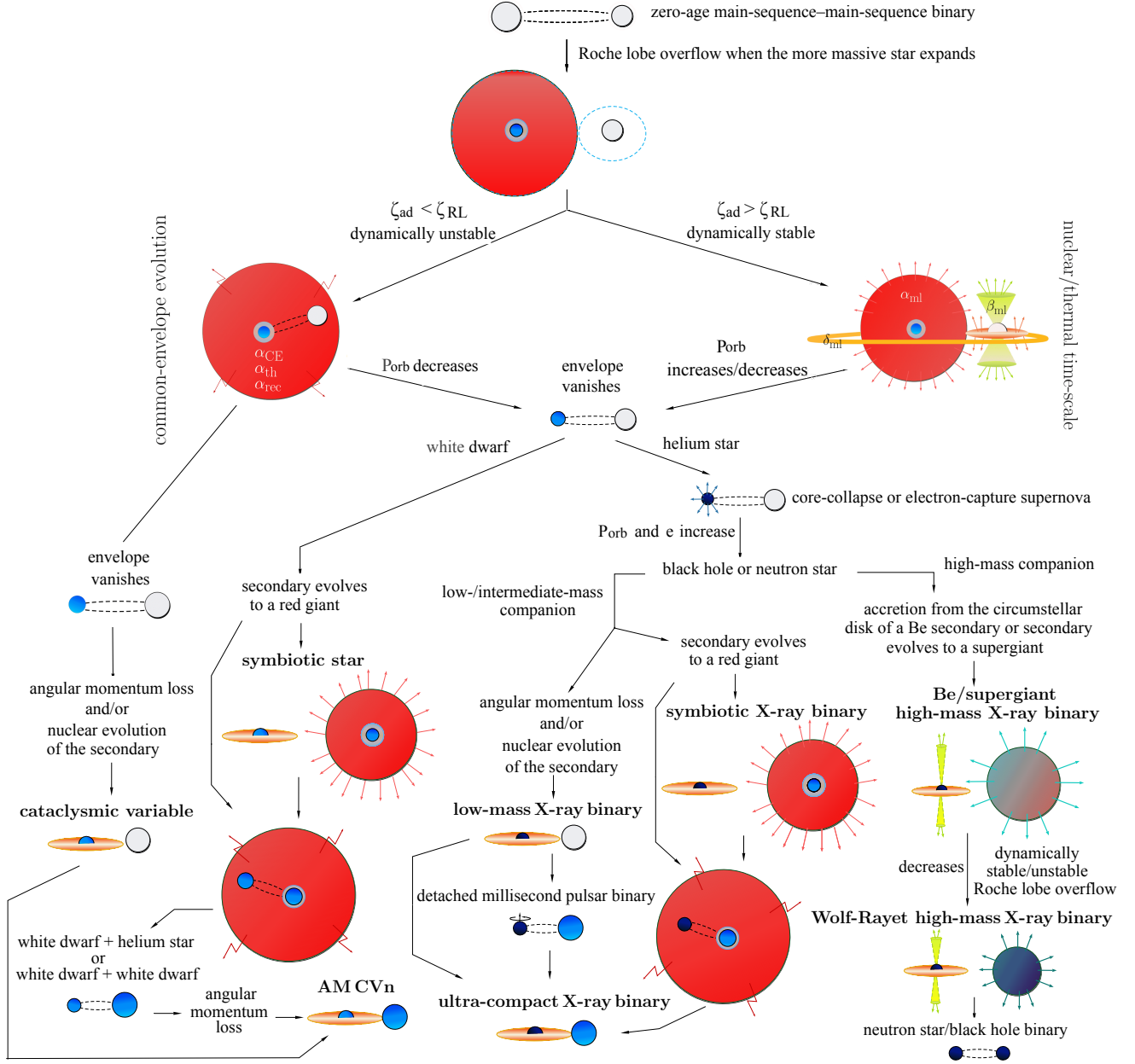


Figure 3.1: Different formation channels of accreting compact object binaries, starting from a zero-age Main-Sequence Binary. On the left branch correspond on the formation path when the compact object correspond to a WD. The diagram shows the path of evolution of a CV, passing from two mass transfer phases, a common envelope phase followed by a dynamically stable mass transfer, would to a double WD binary. This diagram was corrected from Belloni & Schreiber (2023).

Under these conditions, thermal equilibrium can only be maintained if the donor star becomes inflated. As the system evolves, the donor eventually becomes fully convective. This reconfigures the magnetic field, significantly reducing the efficiency of MB. Consequently, the mass transfer rate starts to decrease, and GR becomes the dominant driver of the orbital evolution. This transition typically occurs when the

orbital period reaches ~ 3 h. GR is inefficient at these periods, consequently the mass transfer drops dramatically, leading to a detached phase. The system then enters the period gap. During this detached phase, the orbital period continues to decrease due to GR until it reaches ~ 2 h, at which point the donor refills its Roche lobe and mass transfer restarts. The donor continues to lose mass and at a period of roughly 80 min. becomes a hydrogen-rich degenerate object, evolving from a main-sequence star into a brown dwarf. In response to continued mass loss, the brown dwarf expands, driving the system towards longer orbital periods.

3.2 CV evolution with evolved donors (A-/F-/G-Type)

In the case of a massive donor ($\gtrsim 1 M_{\odot}$), on the MS it possesses a radiative envelope, and therefore MB is not expected to operate, while GR is not efficient at this evolutionary stage. As a consequence, the binary evolution is driven primarily by the nuclear timescale of the donor star. If the orbital separation is sufficiently large, the star can evolve along the MS or even reach the subgiant phase before filling its Roche lobe. During this evolution, the envelope of the star may develop a convective outer layer, allowing MB to become effective. In this case, the onset of mass transfer occurs due to a combination of orbital AML driven by MB and the nuclear expansion of the donor.

If the AML due to MB is dominant the binary evolves toward shorter periods, shrinking the Roche lobe and leading to the onset of mass transfer. At the onset of mass transfer, the donor is significantly more massive than the WD. Therefore, at the beginning of CV evolution, there is a high probability that the mass transfer becomes thermally unstable. The mass transfer rate during this stage can reach values of $\sim 10^{-7}$ – $10^{-6} M_{\odot} \text{ yr}^{-1}$, which are sufficiently high to drive the donor star completely out of thermal equilibrium (Belloni & Schreiber, 2023). As a consequence of the donor’s response to mass loss on the thermal timescale, but also depending on the nuclear evolution and AML, the orbital separation decreases during this phase.

Once the mass transfer stabilizes, the donor star is able to re-establish thermal equilibrium. If the donor star retains a radiative envelope at this stage, the subsequent evolution is driven by the nuclear timescale, leading the system toward longer orbital periods. On the other hand, if orbital AML due to MB or another mechanism is present, the orbital period continues to decrease, and the system follows a CV-like evolutionary path.

In this scenario, the donor star has evolved sufficiently to form a degenerate helium core. During the evolution, continued mass loss from the donor can gradually expose the core and remove most of the hydrogen-rich envelope, eventually leading to the formation of an AM CVn system (Belloni, Diogo & Schreiber, Matthias R., 2023). This evolutionary channel of CVs is one of the proposed pathways for the formation of extremely low-mass WDs (ELM WDs) (Li et al., 2019; El-Badry et al., 2021). Here we test if it can explain the existence of the paradoxical double WD.

Chapter 4

SDSS J1257+5428

The mechanisms leading to the formation of double have attracted significant interest in the last decade, due to the rich physics associated with these binaries. Among them is the system studied in this work, SDSS J1257+5428, which has received particular attention because its properties appear to contradict the expectations from classical binary evolution models.

In this chapter, we provide a detailed overview of the observational history of this object and discuss how its measured parameters establish it as a paradoxical system. Additionally, we review the different evolutionary scenarios that have been proposed in the literature to explain the formation and current configuration of SDSS J1257+5428.

4.1 A brief history of the paradox

SDSS J1257+5428 was discovered by Badenes et al. (2009) as a circular, 4.56 hour-period, single-line, spectroscopic binary from SWARMS (Sloan White dwarf Radial velocity data Mining Survey), from the spectroscopy and photometry data these authors estimate a WD mass $0.92^{+0.28}_{-0.32} M_{\odot}$ and with the orbital parameters conclude an unseen more massive companion than $1.63^{+0.20}_{-0.25} M_{\odot}$ which is in all likelihood either a neutron star or a black hole. Previous work by Kulkarni & van Kerkwijk (2010) and Marsh et al. (2011) using spectroscopy on the R(ed) and B(lue) bands they revisit the properties, and conclude that the system is instead composed of two WDs, a cool extremely low-mass WD and a hotter massive WD. Subsequently, Bours et al. (2015) combined the Ultra-Violet data from *Hubble* Space Telescope with Sloan Digital Sky Survey optical data and refined the parameters of both WDs. They found the cold WD has a mass of $\sim 0.1 - 0.24 M_{\odot}$ and an effective temperature of ~ 6400 K, while the hot companion WD has a mass of $1.06 \pm 0.05 M_{\odot}$ and a effective temperature of ~ 13030 K. The results of this work suggest that the low-mass WD is formed long before the massive hot WD, contrary to what is usually expected. Because of this apparently contradicting properties, the existence of SDSS J1257+5438 has been treated as paradox and indeed it poses an interesting puzzle.

Most recently, Aros-Bunster et al. (2025) reported, based on *Gaia* Data Release 3, that the object 1570271757456852096 is a common proper motion companion to SDSS J1257+5428, making it the third known triple WD system. Using Keck Low Resolution Imaging Spectrometer spectroscopy, *Gaia* and Sloan Digital Sky Survey photometry, together with WD atmosphere models, these authors estimated a cooling age of $\gtrsim 4$ Gyr for the tertiary component. The age of the tertiary provides a potential constraint on the evolutionary history of SDSS J1257+5428. However, this constraint is meaningful only if the tertiary was initially bound to the inner binary; that is, if the triple system formed either primordially or through dynamical three- or four-body interactions. Scattering experiments of binary–binary encounters (Ryu et al., 2017; Barrera Retamal et al., 2024) show that the formation of hierarchical triples is possible with non-negligible probability. Nevertheless, a dynamical formation scenario for SDSS J1257+5428 appears unlikely. The system belongs to the Galactic disk, where stellar densities are too low for frequent four-body interactions to play a significant role (e.g., Binney & Tremaine, 2008). Moreover, the projected separation between the tertiary and the inner binary is 8086 ± 49 au, implying that Zeipel–Lidov–Kozai oscillations would not have significantly affected the evolution of the inner binary. Despite these considerations, the common proper motion is consistent with the tertiary being bound to the inner binary.

4.2 Formation Channels proposed

One of the keys on the formation of close double WDs is that these systems typically exhibit short orbital periods, $\lesssim 1$ d. This indicates that, during their formation, they likely experienced more than one episode of mass transfer, through which the orbital period was significantly reduced. Most likely, this occurred via a common-envelope phase triggered by dynamically unstable mass transfer, followed by a second interaction episode that could take the form of either another common-envelope phase or a dynamically stable mass transfer episode. Considering these evolutionary configurations, several formation channels can be proposed to explain the origin of SDSS J1257+5428.

One channel is considering two episodes of common-envelope each forming a WD, this channel can be ruled out because to form the massive WD ($\sim 1 M_{\odot}$) is necessary a progenitor of star $\sim 5 M_{\odot}$ and it quick evolves, thus this component would be form first. The low-mass cold WD would be originated from a star of $\lesssim 2 M_{\odot}$, which evolution take more time. At the moment of this low-mass cold WD is formed the the age of the massive WD must have been $\gtrsim 1$ Gyr, by the observation the age of the low-mass WD is $\gtrsim 1$ Gyr giving a present of the $\gtrsim 2$ Gyr for the massive WD, additionally combining with the age of the tertiary gives a much older massive WD, which translate on a low effective temperature $\lesssim 10\,000$ K with respect what is observed.

Another formation channel was explored by Aros-Bunster et al. (2025). In this scenario, the low-mass WD forms first through a stable and conservative mass transfer episode, while the massive WD is produced later during a common-envelope phase occurring when the companion reaches the asymptotic giant branch.

However, their analysis shows that, given the upper limits on the initial masses of both stars, it is not possible to accumulate enough mass to produce a viable progenitor for the massive WD.

The only channel left is the one proposed and tested by Belloni et al. (2025). In this scenario, the system first undergoes a common-envelope phase that leads to the formation of the massive WD. This is followed by a dynamically stable mass transfer episode, during which the donor is a star with an evolved core. As discussed in the previous chapter, such stars can host a degenerate helium core, ultimately leading to the formation of an extremely low-mass WD, which corresponds to the low-mass component of the system. During this mass transfer phase, the massive WD accretes material and enters a rejuvenation phase. Belloni et al. (2025) obtained a evolutionary pathways that predicts parameters consistent with those derived from observations of SDSS J157+5428. However, in their model it was necessary to assume an artificially boosted prescription of CARB (Van & Ivanova, 2019) MB. Here we test if assuming saturated MB, as recently suggested for CV evolution, may offer an alternative solution.

Chapter 5

Methods

Given the characteristics of SDSS J1257+5428 that make it a unique and challenging system to explain, the main objective of this work, as stated in the introduction, is to model this system under the assumption that it descends from the evolution of a CV with an evolved-core donor. To achieve this goal, it is necessary to follow the time evolution of the full set of stellar evolution and binary interaction equations, which represents a highly complex task.

For this purpose, we employ the Modules for Experiments in Stellar Astrophysics (MESA) code (Paxton et al., 2011, 2013, 2015, 2018, 2019; Jermyn et al., 2023, version 24.03.1). This code allows us to solve, in one dimension, the time-dependent equations governing stellar structure and composition with high numerical accuracy and physical detail. In addition, MESA is built upon a modular framework, enabling the user to select and control the level of physical detail in the stellar and binary evolution calculations. This modularity also allows the inclusion of external prescriptions beyond the base code, making it possible to modify and explore alternative physical models.

In the fall of this chapter, we describe how the MESA modules were configured to model the stellar evolution of each component and the evolution of the binary system. These configurations closely follow those adopted by Belloni et al. (2025), who generously provided access to the corresponding MESA setup used in that work.

5.1 Stellar Evolution

To model the evolution of the companion star was considered the following parameters. The equation of state is a blend of the OPAL (Rogers & Nayfonov, 2002), SCVH (Saumon et al., 1995), FreeEOS, HELM (Timmes & Swesty, 2000), PC (Potekhin & Chabrier, 2010) and Skye (Jermyn et al., 2021) equations of states. Nuclear reaction rates are a combination of rates from NACRE (Angulo et al., 1999), JINA REACLIB (Potekhin & Chabrier, 2010), plus additional tabulated weak reaction rates (Fuller et al., 1985; Oda et al., 1994; Langanke & Martínez-Pinedo, 2000). Screening is included via the prescription of

Chugunov et al. (2007). Electron conduction opacities are from Cassisi et al. (2007) and the thermal neutrino loss rates are from Itoh et al. (1996) and radiative opacities are primarily from OPAL (Iglesias & Rogers, 1993, 1996), with high-temperature Compton-scattering dominated regime calculated using the equations of Buchler & Yueh (1976).

We adopted a metallicity of $Z = 0.02$ and the grey Eddington $T(\tau)$ relation to calculate the outer boundary conditions of the atmosphere, using a uniform opacity that is iterated to be consistent with the final surface temperature and pressure at the base of the atmosphere. Radiative opacities are taken from Ferguson et al. (2005) for $2.7 \leq \log T \leq 4.5$ and OPAL (Iglesias & Rogers, 1993, 1996) for $3.75 \leq \log T \leq 8.7$. For the evolutionary phases with convective core, that is, core hydrogen and helium, burning, we took into account exponential in the range $1.2 - 2.0 M_{\odot}$ (e.g., Anders & Pedersen, 2023). We assumed that the extent of the overshooting region corresponds to a fraction of H_p (e.g., Schaller et al., 1992; Freytag et al., 1996; Herwig, 2000), with H_p being the pressure scale height at the convective boundary. We further used the nuclear network `cno_extras.net`, which accounts for the nuclear reactions of the carbon-nitrogen-oxygen cycle for hydrogen burning. We allowed the star to lose mass through winds, adopting the Reimers (1975) prescription and setting the wind efficiency to 0.5. We treated convective regions using the scheme by Henyey et al. (1965) for the mixing-length theory, assuming that the mixing length is a factor of H_p (e.g., Joyce & Tayar, 2023). In addition, the boundaries of convective regions are determined using the Schwarzschild criterion. We also included element diffusion due to gravitational settling and chemical and thermal diffusion (Paxton et al., 2015) for ^1H , ^3He , ^4He , ^{12}C , ^{13}C , ^{14}N , ^{16}O , ^{20}N , ^{24}Mg , and ^{40}Ca .

Regarding the WD accretor, we approximated it with a point mass, so we do not compute the detailed structure. We assume a rejuvenation process of the WD accretor that reset to zero at the onset of the detachment (Landin, N. R. et al., 2025)

5.2 Binary Evolution

To configure the binary evolution, we consider the following assumption. The geometry of the binary was defined by the Roche lobe radius of each star, this was computed using the fit of Eggleton (1983). For the mass transfer rate due to the Roche lobe overflow are determined following the prescription of Ritter (1988), in which the atmosphere of the star is filling the Roche lobe, i.e. the mass transfer occurs even if the star radius is smaller than the Roche lobe radius.

The mass transfer due to the Roche lobe overflow was considered as a non-conservative described in Chapter. 2, since we used the formalism of Soberman et al. (1997) to model the mass loss and AML. In the simulation we consider the following sources that we assume that should contribute to this case. The mass lost from the vicinity of the donor star as a fast wind which correspond to a fraction α_{ml} , the isotropic re-emission of the WD accretor correspond to a fraction β_{ml} of the accreted mass (King & Kolb,

1995). For the last, the mass lost from a circumbinary disk correspond to a fraction δ_{ml} that feeds and is modelled as a time-dependent coplanar toroid (Spruit & Taam, 2001). Thus, the accretion efficiency during mass transfer is the given by

$$\epsilon = 1 - \alpha_{\text{ml}} - \beta_{\text{ml}} - \delta_{\text{ml}} \quad (5.1)$$

Given this efficiency the WD accretor can only accrete a part of the mass transferred from its companion. The material accreted can form a hydrogen shell, his time evolution depends on the accretion rate onto the WD accretor. Above of the critical accretion rate the hydrogen shell burning can be stable resulting in an increase in its mass. For this work we used the models from Wolf et al. (2013) to model to determine this critical limit. At lower accretion rate the hydrogen shell can experiment a cycle of nova eruption where mass from the WD accretor is ejected. The amount of accreted mass is not well define and assume that all is expelled is unlikely, we assume that $\beta_{\text{ml}} \leq 85\%$ is ejected. We further assumed that there is a maximum possible accretion rate (Wolf et al., 2013) such that WDs accreting at higher rates will burn stably at this maximum rate and the remaining non-accreted matter will be piled up forming a red-giant-like envelope, which is assumed to be lost from the binary in the form of stellar-like fast winds.

For the AML we took into account that are from de mass loss and the GR with MB since both correspond to the systematic AML.

$$\frac{\dot{J}_{\text{orb}}}{J_{\text{orb}}} = \frac{\dot{J}_{\text{ML}}}{J_{\text{orb}}} + \frac{\dot{J}_{\text{CB}}}{J_{\text{orb}}} + \frac{\dot{J}_{\text{GR}}}{J_{\text{orb}}} + \frac{\dot{J}_{\text{MB}}}{J_{\text{orb}}} \quad (5.2)$$

where

$$J_{\text{orb}} = M_{\text{WD}} M_2 \sqrt{\frac{G a}{M_{\text{WD}} + M_2}} \quad (5.3)$$

where G is the gravitational constant, M_{WD} is the mass of the WD accretor, M_2 is the mass of the donor star. The AML as a consequence of the mass loss from the the WD accretor and the donor star is

$$\frac{\dot{J}_{\text{ML}}}{J_{\text{orb}}} = \left(\beta_{\text{ml}} \frac{q^2}{1+q} + \alpha_{\text{ml}} \frac{1}{1+q} \right) \frac{\dot{M}_2}{M_2} \quad (5.4)$$

where $q = M_2/M_{\text{WD}}$. The AML produce by the circumbinary disk has a form

$$\frac{\dot{J}_{\text{CB}}}{J_{\text{orb}}} = \gamma \delta_{\text{ml}} (1+q) \frac{\dot{M}_2}{M_2} \quad (5.5)$$

where

$$\gamma = 1.3 (r_i/a)^{1/2} (t_*/t_{\text{vi}})^{1/2} \quad (5.6)$$

$\delta_{\text{ml}} = 0.1\%$, $t_{\text{vi}} = 100$ yr is the disk viscous timescale, and t_* is the time in yr the onset of the mass transfer. The binary initially hosts an empty circumbinary disk, which is fed at a rate corresponding to a fraction δ_{ml} of the mass transfer rate lost through the outer Lagrangian point L_2 . The inner disk radius, r_i , is assumed

to be 1.7 times the orbital separation of the binary. This progressive buildup of the disk mass implies a growing torque during the CV evolution. For the orbital decay due to the emission of gravitational waves is given by

$$\dot{J}_{\text{GR}} = -\frac{32}{5c^5} \left(\frac{2\pi G}{P_{\text{orb}}} \right)^{7/3} \frac{(M_1 M_2)^2}{(M_1 + M_2)^{2/3}} \quad (5.7)$$

For the MB as mentioned above we adopted a different prescription used in Belloni et al. (2025) where was adopted a boosted version of the CARB prescription (Van & Ivanova, 2019) ranging the boost factor from 1 – 100. In this work we used a prescription of a saturated and boosted MB which is based on the saturated, boosted and disrupted (SBD) MB from Belloni et al. (2023) used to explain WD plus K-dwarfs, and explain standard CVs Barraza-Jorquera et al. (2025). The base model in this prescription was the magnetic braking model from Matt et al. (2015) which was re calibrated.

$$j_{\text{MB,SAT}} = K \begin{cases} T_0 \left(\frac{P_{\text{rot}}}{1\text{d}} \right)^{-3} \left(\frac{\tau_c}{\tau_{c,\odot}} \right)^{-2} \left(\frac{R}{R_\odot} \right)^{3.1} \left(\frac{M}{M_\odot} \right)^{0.5} & P_{\text{rot}} \geq P_{\text{crit}} \\ T_0 \left(\frac{P_{\text{rot}}}{1\text{d}} \right)^{-1} \left(\frac{P_{\text{crit}}}{1\text{d}} \right)^{-2} \left(\frac{\tau_c}{\tau_{c,\odot}} \right)^{-2} \left(\frac{R}{R_\odot} \right)^{3.1} \left(\frac{M}{M_\odot} \right)^{0.5} & P_{\text{rot}} < P_{\text{crit}} \end{cases} \quad (5.8)$$

Where K is the boosting factor, $T_0 = -2.816 \times 10^{31}$ is a calibration parameter and $\tau_{c,\odot} = 34.869$ d is the convective turnover time of the sun. For the saturation threshold we adopt $P_{\text{crit}} = 0.04\tau_c$ where τ_c is the convective turnover time which was calculated with the same way as in Van & Ivanova (2019)

$$\tau_c = \int_{R_b}^{R_*} v_c^{-1} dr \quad (5.9)$$

where R_* is the stellar radius, R_b is the radial position of the base of the convective zone and v_c is the convective velocity. The re-calibration for the MB are from Barraza-Jorquera & Schreiber et al, (in prep), where used same base of MB with a SBD prescription to model standard CVs and AM CVn binaries.

MB was assumed to be triggered only during evolutionary phases in which the donor has a non-negligible convective envelope enclosing a radiative core. To implement this condition, we required the mass fraction of the convective envelope (q_{conv}) to exceed 0.03.

5.3 Searching a model for SDSS J1257+5428

Having established the configuration to evolve both stars and the assumptions adopted for the evolution of the binary system, we proceeded to search for a model whose evolutionary path reproduces a system similar to SDSS J1257+5428, that is, one in which the parameters of the secondary star converge to the observed properties of the extremely low-mass WD. To perform the search for the best-fitting model, we considered as a starting point the initial parameters of the best model found by Belloni et al. (2025), and varied only the initial orbital period (1–10 d), under the assumption of synchronization between the

orbital period and the stellar rotation, the mass of the accreting WD ($0.65\text{--}1.05\text{ M}_{\odot}$), and the mass of the companion ($1.05\text{--}1.30\text{ M}_{\odot}$).

The following parameters were fixed to the values corresponding to the reference model: the fraction of mass lost from the donor ($\alpha_{\text{ml}} = 15\%$), the mixing length ($2.5, H_p$), and the overshooting extension ($0.060, H_p$), where H_p corresponds to the pressure scale height at the convective boundary. Regarding the boosting factor of MB, we prioritized a value of $K = 80$, since this allows us to use the same prescription used for modelling standard CVs and AM CVn binaries and expect a no need of re-boost the MB. In all models, we assumed a solar metallicity ($Z = 0.02$) and circular orbits

Chapter 6

Results

We computed several evolutionary tracks exploring the range of initial parameters described in the previous chapter. The best-fit model was identified as the sequence whose evolutionary path simultaneously reaches the observed effective temperature and orbital period while reproducing the component masses within their observational uncertainties. The initial parameters varied to obtain this model are $P_{\text{orb}} = 8.147$ d, $M_{\text{WD}} = 0.71 M_{\odot}$, and $M_d = 1.3 M_{\odot}$. The final properties predicted by the model are presented in Table 6.1, while the evolution of the main system properties is shown in Figures 6.1, 6.2, 6.3, 6.4 and 6.5.

Table 6.1: Predicted by the model and observed (Bours et al., 2015; Aros-Bunster et al., 2025) parameters of SDSS J1257+5428.

Parameter	Observed	Observed (fixed $\log g = 6$)	Observed (fixed $\log g = 7$)	Predicted
P_{orb} (h)	4.56	4.56	4.56	4.48
Total age (Gyr)	$\gtrsim 4$	$\gtrsim 4$	$\gtrsim 4$	5.94
Hot WD				
M ($M_{\odot}$)	1.06 ± 0.05	1.04 ± 0.05	1.00 ± 0.05	1.05
cooling age (Gyr)	1.0 ± 0.1	1.0 ± 0.1	0.9 ± 0.1	0.87
Cold WD				
T_{eff} (K)	6400 ± 87	6395 ± 80	6460 ± 80	6386.35
M ($M_{\odot}$)	$\lesssim 0.24$	$\lesssim 0.24$	$\lesssim 0.24$	0.18
$\log g$ (cm s^{-2})	5.26	6.0	7.0	6.82
R ($R_{\odot}$)	0.032 ± 0.003			0.027
cooling age (Gyr) since The onset of the detachment				0.87

The model starts with a binary system composed of a WD and a MS star. During the initial stages, the MS star has a convective core and a radiative envelope. Only a very thin outer convective layer is present, which is negligible; therefore, MB is not active. In this phase, the binary evolves toward longer orbital periods due to mass loss driven by stellar winds from the MS star, which evolves as an isolated star.

This detached evolutionary phase of the MS star is shown in the Hertzsprung–Russell diagram in Figure 6.1, where the end point is marked by a green star symbol. During this stage, the star begins to develop a helium core (at 4675.93 Myr). In addition, the convective layer in the envelope becomes non-negligible ($0.3 \lesssim q_{\text{conv}}$), resulting in a stellar structure characterized by a convective envelope and a radiative core.

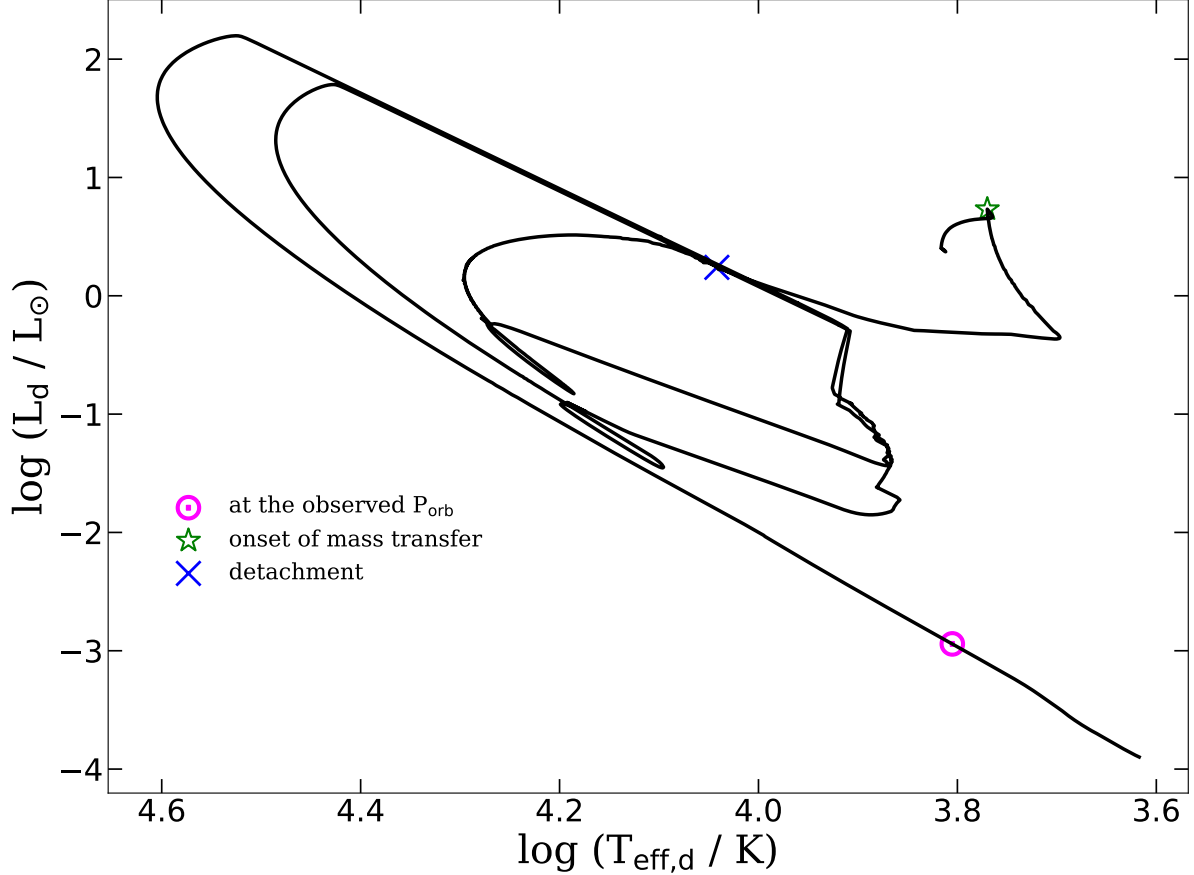


Figure 6.1: The green stars mark the onset of mass transfer. The blue crosses indicate the end of the CV phase and the detachment of the binary. The magenta solar symbols highlight the evolutionary stage at which the effective temperature of the model matches the observed value of the low-mass WD in SDSS J1257+5428. This track illustrates the evolution of the donor star, from its detached MS phase prior to mass transfer, through the CV phase, and after binary detachment up to the cooling sequence of the proto-WD.

Consequently, the condition for MB is satisfied, and from this point onward MB is able to extract orbital angular momentum from the system. As a result, the binary evolves toward shorter orbital periods, with an AML rate of the order of $\sim 10^{34}$ to $\sim 10^{36}$ dyn cm. As the binary shrinks, the donor continues its evolution until reaching the beginning of the subgiant phase, at which point it fills its Roche lobe at an orbital period of ~ 23.26 h (at an age of 4988.98 Myr).

At the onset of mass transfer, the star possesses a helium core of $0.177 M_{\odot}$, initiating a CV evolution

with an evolved donor at an orbital period of ~ 23.26 h. The evolution of the mass transfer rate and the mechanisms driving the orbital AML are shown in Figure 6.2. At this stage, MB is sufficiently strong to drive a high mass transfer rate $\sim 10^{-7} M_{\odot} \text{ yr}^{-1}$, satisfying the condition for stable hydrogen burning. This allows the accreting WD to increase its mass, a process we assume leads to rejuvenation. On the other hand, the high mass transfer rate perturbs the thermal equilibrium of the donor; nevertheless, due to the high efficiency with which orbital angular momentum is removed by MB, CAML, and the circumbinary disk, the mass transfer evolution is dominated by the AML timescale.

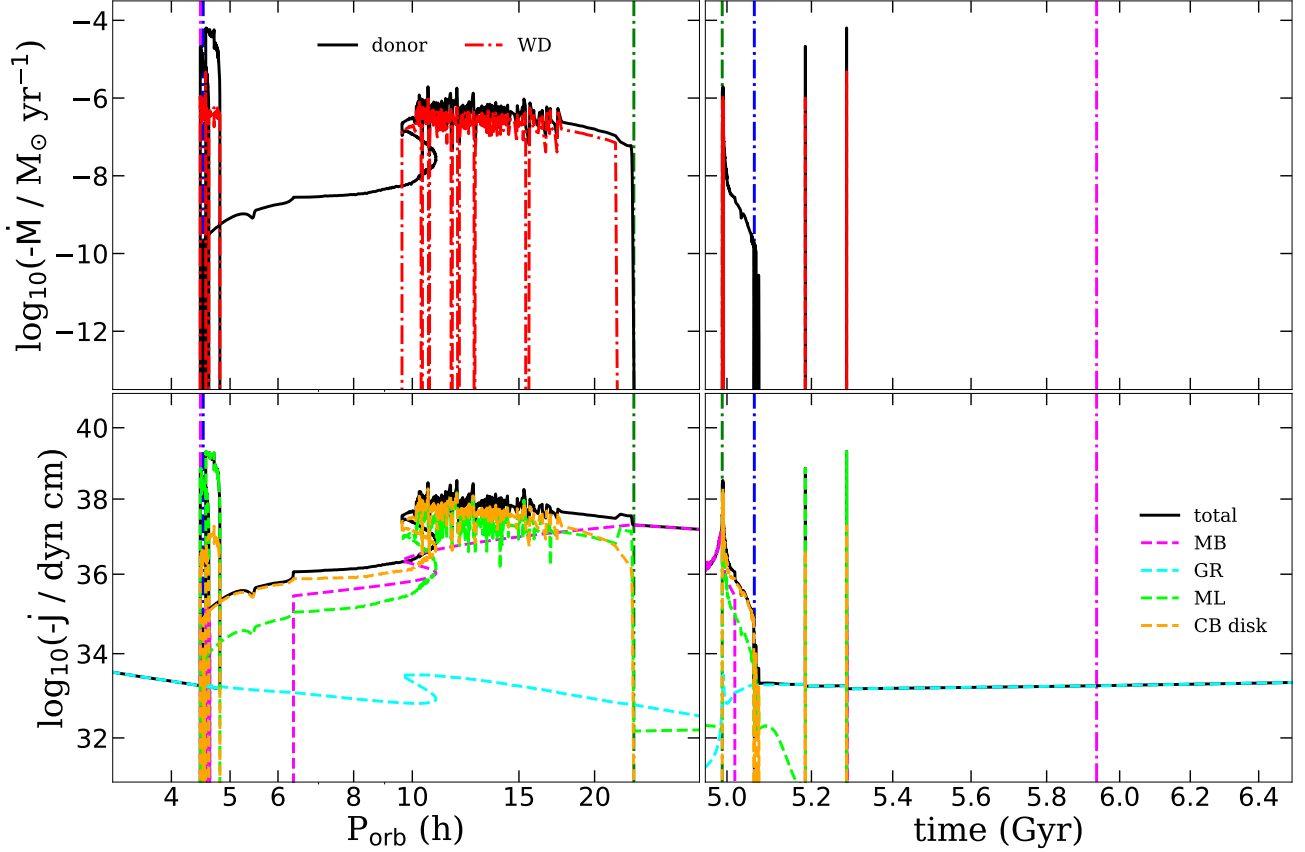


Figure 6.2: **Top panels:** Evolution of the mass transfer rate and accretion rate as a function of orbital period and time. **Bottom panels:** Evolution of the total AML and its individual sources as a function of orbital period and time. The vertical lines indicate different evolutionary phases of the binary: the green line marks the onset of mass transfer, the blue line indicates the time and orbital period at which the binary detaches, and the magenta line corresponds to the moment when the model reaches the observed effective temperature of the low-mass WD.

During this phase, the donor radius decreases while the surface gravity ($\log g$) increases, whereas the effective temperature remains approximately constant, as shown in Figure 6.4 and 6.3. After the point marking the onset of Roche lobe overflow, both the donor radius and the orbital period start to decrease. Upon reaching orbital periods of ~ 15 – 18 h, MB ceases to dominate the evolution, and the torques exerted

by the circumbinary disk become the main driver of the mass transfer. When the system reaches an orbital period of ~ 10.01 h, the donor expands in order to re-establish thermal equilibrium, causing a temporary increase in the orbital period.

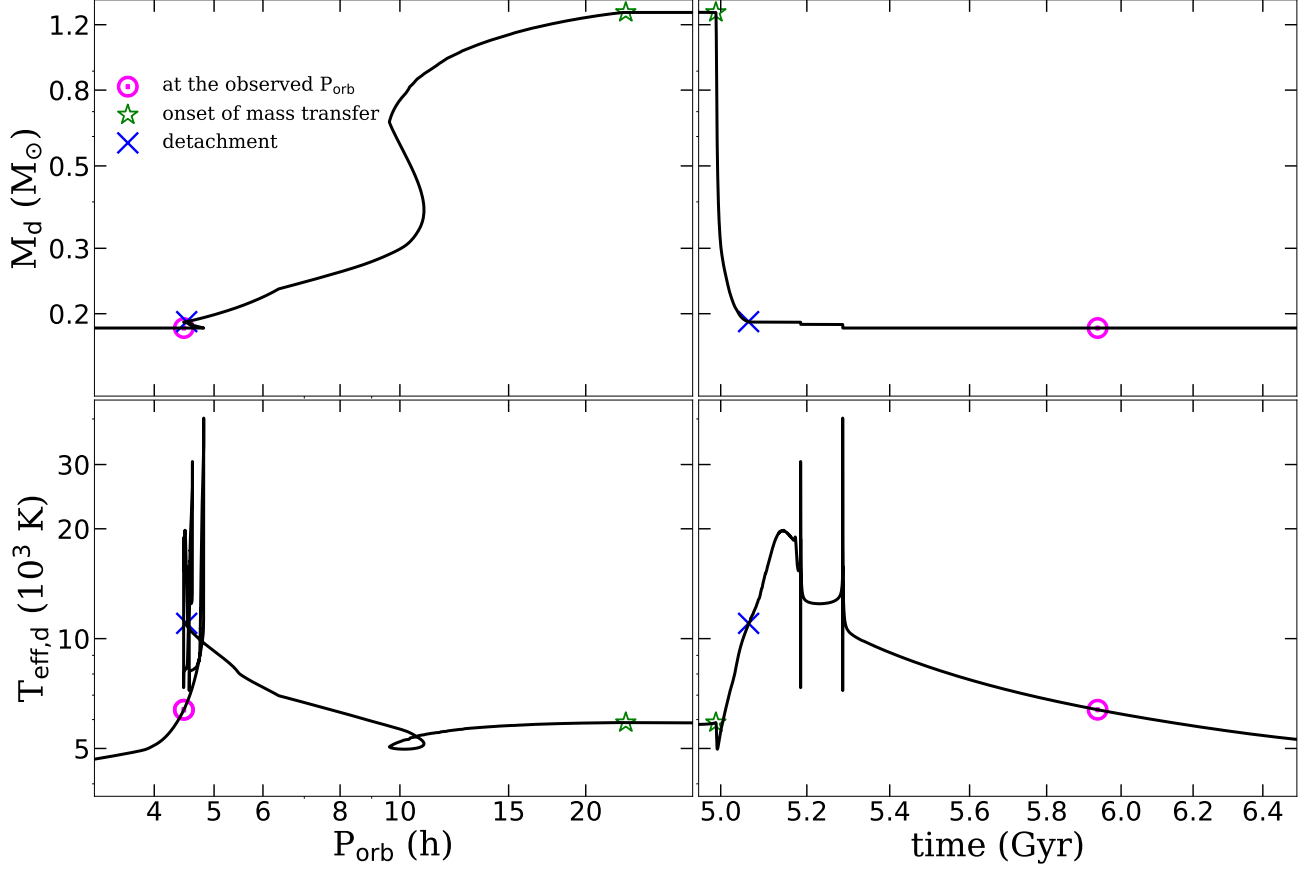


Figure 6.3: **Top panels:** Evolution of the donor mass as a function of orbital period and time. **Bottom panels:** Evolution of the donor effective temperature as a function of orbital period and time. The symbols have the same meaning as in Figure 6.1. This figure illustrates the evolution of these parameters during the CV phase and after the detachment of the binary.

This behaviour can be seen in the upper-left panel of Figure 6.4, where for a short interval the decrease in the donor radius becomes less pronounced. However, since mass transfer still proceeds on the AML timescale, the system subsequently evolves again toward shorter periods. Additionally at this moment the effective temperature of the donor heats up.

At this stage, the mass transfer rate no longer exceeds the stable hydrogen-burning limit, and therefore the accreting WD stops increasing its mass, reaching a final mass of $1.05 M_\odot$. This mass is maintained due to the occurrence of nova eruptions, which expel a large fraction of the accreted material from this point onward. The bottom panels of Figure 6.5 show the evolution of the accreting WD mass as a function of orbital period and time, where it is possible to clearly identify the point at which the stable

hydrogen-burning limit ceases to be satisfied. The upper panels of the same figure display the evolution of the donor's convective envelope mass.

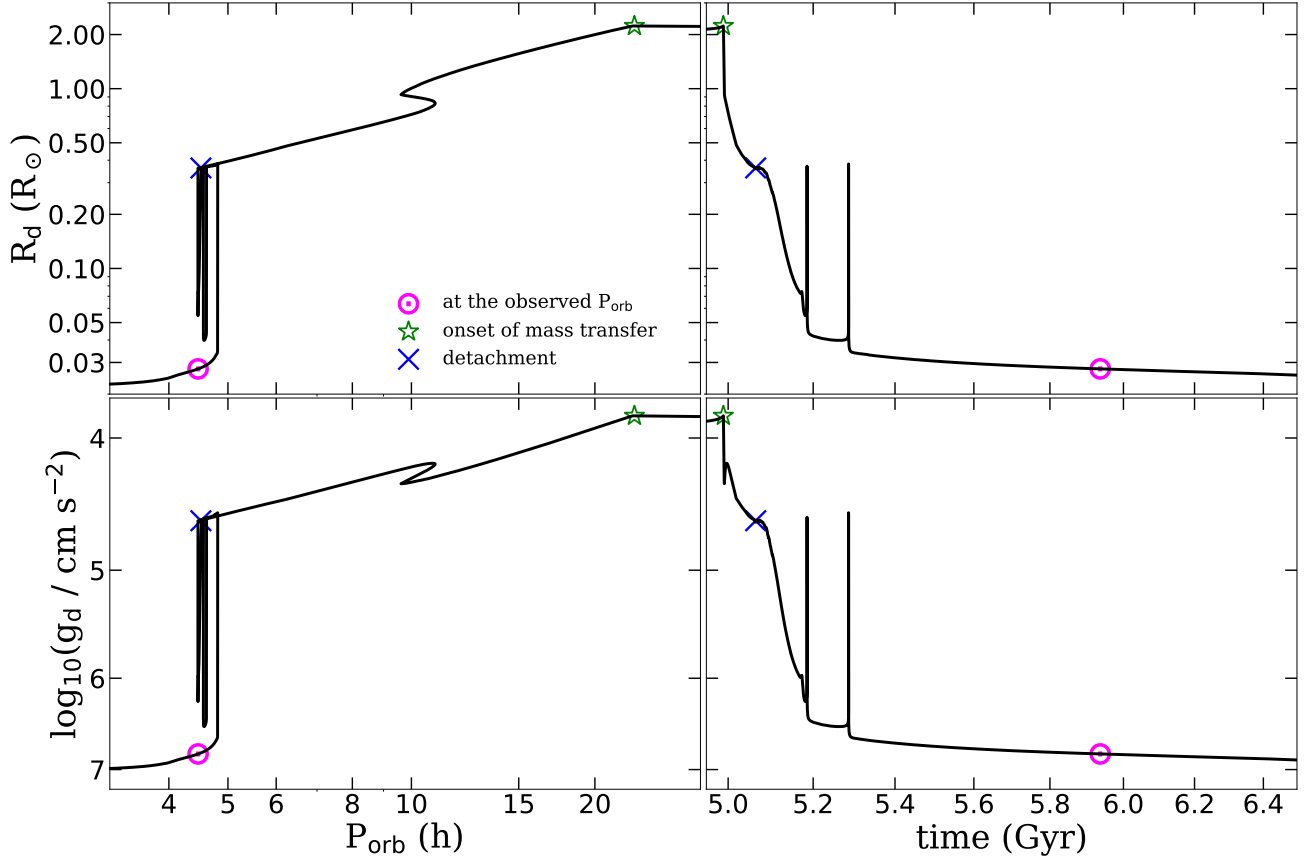


Figure 6.4: **Top panels:** Evolution of the donor radius as a function of orbital period and time. **Bottom panels:** Evolution of the donor surface gravity ($\log g$) as a function of orbital period and time. The symbols have the same meaning as in Figure 6.1. These tracks illustrate the contraction of the donor and the increase in surface gravity as it evolves toward a degenerate stellar object.

The continued mass loss causes the donor's convective envelope to progressively shrink until it becomes negligible, such that the condition for MB is no longer satisfied and this mechanism starts to decay (at ~ 5020.3 Myr). As a consequence, the accretion rate decreases, although not abruptly.

Figure 6.2 illustrates how the high efficiency of CAML mechanisms (green and gold curves), in particular the circumbinary disk, is able to sustain mass transfer for a significant period of time. The strong AML associated with the circumbinary disk allows the system to continue evolving toward shorter orbital periods despite the decay of MB.

Eventually, when an orbital period of ~ 4.51 h is reached, the mass transfer rate drops completely, leading to the detachment of the binary (at 5062.62 Myr). At this stage of the evolution, the donor has become an extremely low-mass proto-WD, with a mass of $\approx 0.19 M_\odot$, a hydrogen-rich envelope mass of $\sim 0.04 M_\odot$, and an effective temperature of 11 011.33 K.

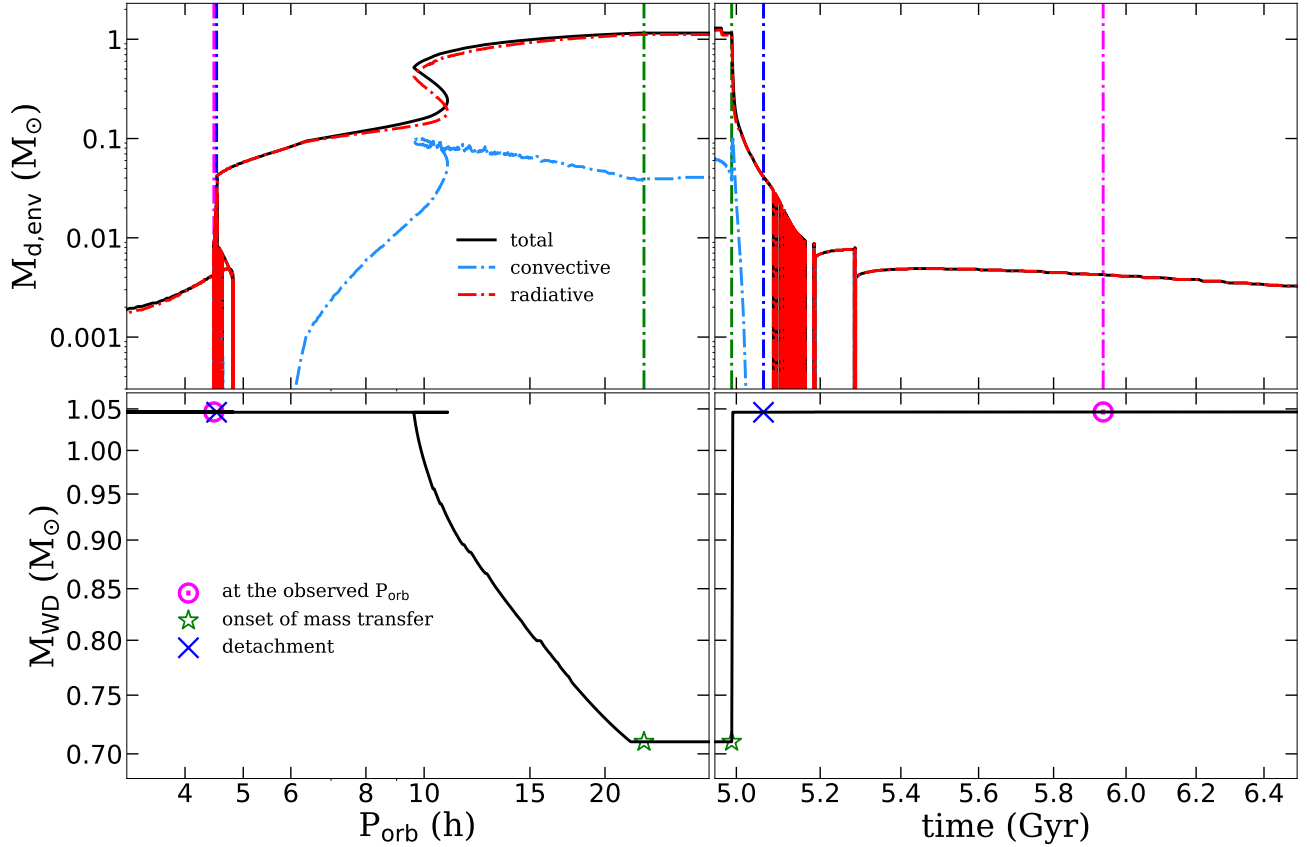


Figure 6.5: **Top panels:** Evolution of the donor total envelope mass, as well as its convective and radiative envelope components, as a function of orbital period and time. **Bottom panels:** Evolution of the WD mass as a function of orbital period and time. The symbols have the same meaning as in Figure 6.1. The vertical lines have the same meaning as in Figure 6.2. These tracks illustrate the structural evolution of both components during the CV phase and after the final detachment of the system.

The proto-WD undergoes a series of hydrogen flashes in its envelope over a timescale of approximately ~ 340 Myr. These episodes can be identified in Figure 6.1, where the evolutionary track displays the characteristic loops. During this phase, the star experiences strong structural changes, with the effective temperature increasing dramatically and reaching values above $\sim 20\,000$ K.

During each flash, a fraction of the hydrogen envelope mass is ejected, producing a slight increase in the orbital period. At the same time, the proto-WD experiences a pronounced expansion in radius, which leads to a marked increase in luminosity. That can be seen in Figures 6.3 and 6.4

After completing these episodes, at an age of 5285.59 Myr, the hydrogen envelope is entirely removed, leaving the degenerate helium core exposed, that is, a helium WD (HeWD). This object subsequently enters the cooling sequence at approximately ~ 5936.14 Myr. From this point onward, the evolution of the system is driven exclusively by AML due to GR, leading the binary toward shorter orbital periods.

At an age of 5936.14 Myr, the HeWD reaches an effective temperature comparable to the observed value, with a corresponding orbital period of 4.48 h. At this stage, the evolutionary path of the model

reproduces the observed properties of the system in a consistent manner marked with the solar symbols on the different panels of the Figures 6.3,6.4 and 6.5.

Chapter 7

Discussion

Along the evolutionary track of our best-fitting model, we have shown in the previous chapter that the evolution of a CV with an evolved donor predicts the observed parameters at the moment of observation. In our binary configuration, we adopted a standard MB prescription based on the Matt framework, using the same calibrations commonly employed to model standard CVs and AM CVn systems. Within this framework, we find that SDSS J1257+5428 can be reproduced without the need for introducing any additional scaling factors.

This result contrasts with the work of Belloni et al. (2025), who required an extra multiplicative factor of 100 in their AML prescription in order to model SDSS J1257+5428. It is also worth noting that we do not include a MB disruption in our model, as the donor star never becomes fully convective. Consequently, when the donor develops only a negligible convective envelope, MB shuts off, producing a sharp decline.

Nevertheless, the dominant driver of the system’s evolution is a strong CAML mediated by a circumbinary disk. This mechanism significantly enhances the mass transfer rate, leading to a higher accretion rate onto the WD. As a result, the accretor can increase its mass while simultaneously experiencing efficient orbital AML. In the absence of this mechanism, the system would tend to diverge due to the donor’s thermal response (or detach at a much longer period). During the late mass transfer phase, the evolution must be driven by the circumbinary disk, as it constitutes the most efficient CAML mechanism capable of shrinking the orbital period to the observed value of SDSS J1257+5428 prior to detachment.

One of the critical assumptions in our modeling concerns the rejuvenation process of the accreting WD. Following the mass transfer episode, the cooling age of the accretor is effectively reset. In our model, the WD mass increases due to stable hydrogen burning until a limiting value is reached, after which the accreted material is expelled through isotropic re-emission. During this phase, it is unlikely that the internal structure of the WD remains unchanged. Material impacting the surface of the accretor can induce compression and lead to an increase in temperature (Epelstain et al., 2007; Townsley & Bildsten, 2004; Townsley & Gänsicke, 2009). This process, however, is in general not yet fully understood and,

in particular, not modeled in our approach. We simply assume that the accreting WD is heated up sufficiently to match the observations.

As discussed both above and in the previous chapter, the circumbinary disk plays a crucial role in the evolution of the CV. This mechanism is neither intrinsic to the stellar components nor required to model standard CVs using the SBD MB prescriptions, which are able to reproduce observations without invoking this AML channel. This places our scenario in a somewhat unfavorable context, as it requires special conditions, such as the presence of surrounding material capable of forming a circumbinary disk, which then dominates the evolution and allows the model to reproduce SDSS J1257+5428.

Despite these caveats, obtaining a self-consistent evolutionary model provides valuable insight into the nature of these systems, which involve extreme physical processes relevant to SN Ia progenitors. After the evolutionary track of our model passes through the observed parameters of SDSS J1257+5428, the system continues to evolve toward shorter orbital periods driven by GR. At an orbital period of 0.0028 d, the HeWD fills its Roche lobe, triggering a third episode of mass transfer. At the onset of this phase, the mass transfer becomes dynamically unstable. Given that both components are degenerate, the system does not undergo a common-envelope phase, but instead potentially evolves into a dynamically driven double degenerate double detonation SN Ia.

This scenario represents one of the possible alternative channels for producing SN Ia (Liu et al., 2023). If the He WD were to survive the supernova explosion, the extremely short orbital period would result in the remnant being ejected as a hypervelocity star, as proposed by Shen et al. (2018). The ejection velocity can be approximated using the final orbital velocity in our model, $v = 1373.68 \text{ km s}^{-1}$, which lies within the range of velocities observed for hypervelocity stars (Shen et al., 2018; El-Badry et al., 2023). In this way, double WD systems not only provide insight into the progenitors of SN Ia, but may also offer an explanation for the origin of hypervelocity stars in our Galaxy.

Chapter 8

Conclusions

In this work, we performed binary evolution models using the MESA code to model the system J1257+5428 through the CV formation channel. Saturated and boosted MB provides the appropriate angular momentum loss required for the donor star to enter the CV phase with evolved nuclear donor. In the best-fitting model, at the end of the CV phase, the secondary star fully converges to the observed parameters of the extremely low-mass WD, while the WD primary increases its mass to the observed value. The only discrepancy found in this model concerns the predicted orbital period, which is slightly shorter than the observed one.

We adopted the MB prescription proposed by Matt et al. (2015), with the re-calibration and boosting factor from Barraza-Jorquera & Schreiber et al, (in prep). This re-calibrated prescription can reproduce the evolution of CVs and potentially explain the formation of AM CVn systems through the CV channel. During the CV evolution, the main driver was the circumbinary disk, an initially unexpected behaviour. The system can be explained using this MB prescription without any further fine-tuning.

The possibility of forming a double WD binary through the evolution of a CV highlights the important role that these systems play. In this way, CVs provide a framework to study in future work the phenomena associated with double WD systems including double detonation SN Ia explosions.

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