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COMPARACIÓN DE ESTRATEGIAS DE RESOLUCIÓN DE PROBLEMAS DE CONTROL PREDICTIVO BASADO EN MODELOS ECONÓMICOS EN UN SISTEMA EXPERIMENTAL

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Comparación de Estrategias de Resolución de Problemas de Control Predictivo basado en Modelos Económicos en un Sistema Experimental

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Resumen

Los sistemas de toma de decisiones para los procesos industriales representan un importante desafío para los algoritmos de optimización existentes, considerando distintas escalas de producción, y distintas frecuencias de actualización. Para abordar de manera adecuada el problema de toma de decisiones, la literatura se propone la separación de los problemas de acuerdo a su objetivo y a su frecuencia de actualización.

El problema relacionado con la toma de decisiones a escala operacional, entendido como la actualización y seguimiento de consignas de control utilizando criterios económicos, se ha abordado separándolos en una capa de optimización económica estacionaria (optimización en tiempo real, RTO), que actualiza las consignas de un sistema de control basado en modelos (control predictivo, MPC), que a su vez actúa sobre el control regulatorio que se encarga de rechazar perturbaciones. Si bien mediante esta metodología es posible abordar en tiempos razonables problemas de gran escala, la diferencia entre los modelos utilizados en las capas de RTO y MPC puede provocar problemas de convergencia y disminución en la optimalidad alcanzada por el proceso. Para solucionar este problema, se ha propuesto unir las capas de RTO y MPC en un sistema de control con objetivo económico (EMPC), que utilice un modelo dinámico generalizable para calcular trayectorias del proceso con objetivos económicos. Esta formulación tiene una ventaja adicional, relacionada con utilizar los grados de libertad dinámicos en beneficio de la economía del proceso. Sin embargo, cuando se aplica a procesos con una gran cantidad de variables/unidades, la velocidad de resolución del problema de optimización puede verse afectada, a tal punto de hacer la solución inaplicable.

Actualmente existe una sección en la ingeniería aplicada al control de procesos industriales en la que no se pueden utilizar las estrategias usuales de resolución de problemas de optimización en tiempo real o con una frecuencia de actualización de decisiones (o entrega de soluciones) tan rápida como se necesita. Básicamente, existe una cantidad de problemas que no pueden ser resueltos en un periodo de tiempo que requiere alguna de las capas de optimización y a este tipo de problemas se les llama de gran escala.

En este trabajo se presentan dos problemas históricos, los que resultan ser de gran escala al intentar resolverlos. El primero es el problema de redes de distribución de recursos compartidos, en el que

distintas unidades de proceso compiten por el uso de recursos para lograr llegar a consignas impuestas por capas superiores. El segundo problema es uno de optimización estocástica en dos etapas, donde la calidad de la materia prima de una unidad de proceso es desconocida, pero se conoce información estadística-histórica acerca de esta.

Para tratar estos problemas se propone la comparación de tres metodologías, dos de las cuales modifican el enfoque jerárquico usual y agregan una capa de optimización extra, mientras que la tercera intenta resolver el problema en su forma original. Las primeras dos metodologías, llamadas clipping y Lagrangiano aumentado, modifican el problema transformándolo en una cantidad mayor de problemas más pequeños (o que consideran menos información), los que son resueltos de forma iterativa, controlados por la capa de coordinación para asegurar su convergencia. La tercera metodología se basa en el uso del operador matemático de "proyecciones" intercalado con pasos de gradiente, los que bajo restricciones controladas prometen la convergencia de los resultados.

Adicionalmente, se implementan en un sistema experimental con el fin de comparar el desempeño de una rutina de control. Con esto se pudo comprobar que el uso de las metodologías propuestas en este trabajo asegura la reducción de los tiempos de cálculo entre 40% y un 90%, comparado con el caso original. Se pudo comprobar, además, que en general el uso del método de proyecciones entregó los mejores resultados, asegurando una disminución del tiempo de cálculo significativa en comparación al caso original y al caso coordinado.

La disminución alcanzada en el tiempo de cálculo es una oportunidad para poder implementar rutinas de control óptimo en plantas industriales más complejas e interconectadas, con modelos fenomenológicos más exactos, con frecuencias de actualización más cortas o con restricciones operacionales cada vez más reales.

Palabras clave: Problemas de gran escala, Distribución de recursos, Optimización estocástica, Sistema benchmark, Capa de coordinación, Clipping, Lagrangiano aumentado, Proyecciones.

Abstract

Decision-making systems for industrial processes represent a major challenge for existing optimization algorithms. They consider different production scales and different update frequencies. In order to adequately address the decision-making problem, the literature proposes the separation of the problems according to their objective and its updating frequency. The problem related to decision-making at operational level, understood as the updating and monitoring of control set-points using economic criteria, has been addressed by separating them into a layer of stationary economic optimization (real-time optimization, RTO).

RTO works updating the set-points of a control system based on models (predictive control, MPC). This layer acts on the regulatory control, which is responsible for rejecting disturbances. Although this methodology makes possible to address large-scale problems in reasonable time, the difference between the models used in the RTO and MPC layers can cause problems of convergence and reduce the optimality achieved by the process. To solve this particular problem, it has been proposed to link the RTO and MPC layers in an economic objective control system (EMPC), which uses a generalizable dynamic model to calculate trajectories of the process with economic objectives. This formulation has an additional advantage, related to using dynamic degrees of freedom for the economic benefit of the process. However, when applied to processes with a large number of variables/units, the calculation time of the optimization problem may be affected, to the point of making the solution inapplicable.

Industrial process control is based on formulating and solving optimization problems, which are generally arranged in hierarchical or layered structures, each layer has different goals and try to comply different tasks. An intuitive connection exists between layers: the upper ones -or those that handle more information- give instructions to the lower ones, which use a higher frequency of information updating in comparison, to fulfill the instructions.

When formulating the proper optimization problems, is common to use an economic criterion while trying to measure the performance of the decision being calculated. Considering this, comparing profits and expenses in the objective functions of the problems seems to be intuitive.

Nowadays, there is an area in control process engineering in which the current problem-solving strategies cannot be used in real time or with a desired actualization frequency. Basically, there are

a number of problems that cannot be solved in a required period of time by any of the optimization layers. This type of problems are called large-scale problems.

This work deals with two historical large-scale problems. The first one is the shared resources and distribution problem, where different process units compete for the use of different resources while trying to comply tasks imposed by higher layers. The second problem is the two-stage stochastic optimization, where the quality of a process input is unknown, but historical-statistical information is available about its behavior.

To deal with these problems, the comparison of three methodologies is proposed. Two of them works by modifying the hierarchical structure and inserting an extra optimization layer called coordinator and, also, modifying the topology of the original optimization problem by breaking it into a larger amount of smaller problems. These smaller problems will be solved in an iterative manner in order to guarantee the convergence of the method, controlled by the coordinator. These methodologies are called “clipping” and “augmented Lagrangean”. The third methodology attempt to solve the original problem by using the projection operator alternated with gradient steps. This method has a properly studied convergence under controlled parameters.

Additionally, the use of an experimental system is proposed to implement and compare the performance of a control routine. In this system was possible to verify that the use of the methodologies proposed in this work ensures the reduction of calculation times between a 40% and 90%, compared to the original case. It was also possible to verify that, in general, the use of the projection method delivered the best results, ensuring a significant reduction in the calculation time compared to the original case and the coordinated case.

The decrease reached in the calculation time is an opportunity to implement optimal control routines in more complex and interconnected industrial plants with: (i) more accurate phenomenological models, (ii) shorter update frequencies or (iii) more invasive restrictions.

Key words: Large-scale problems, Shared resources distribution, Stochastic optimization, Benchmark system, Coordination layer, Clipping, Augmented Lagrangean, Projections.

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Dedicado a la memoria de

Sebita Muñoz

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1. Introduction

In chemical engineering, the goal of process design is the creation or modification of a "flowsheet" which is able of manufacturing the desired product (Westerberg, 1997). It may be obvious to think that the style of designing has evolved since his beginning in the industrial revolution, due to advances in technology and engineering. Bearing in mind that the ultimate goal of almost every company is to generate profits by relying on the transformation of the raw material into a product, a wide range of variables must be considered when designing and operating, since the profit ratio is a function of them (Kumcu et al., 2003): in one hand, you can work internally on the production process itself, and on the other hand, you should consider that the sale of the product is immersed in external frameworks, be they legal, environmental, quality, economic, etc., which should comply so the product can effectively be traded. Generally, there is no control over these aspects, which, are highly dynamic over time and always tend to be stricter; only a versatile and robust design and operation is able to overcome the requirements of optimizing the process internally or to work against changes in any external framework that applies to the industry at issue.

Since industrial development involves searching for comparative advantages with respect to the market, the idea of producing with the minimum amount of resources, be they energy or raw materials, seems to be the intuitive next step. Optimization is one of the most versatile tools spared in process engineering and its application is based on the proper formulation and resolution of an optimization problem which can take into account different objectives. In chemical engineering, there is always room for improvement in process operations because it is unlikely for any process to operate at the theoretically global optimal conditions for any substantial length of time (Ellis et al., 2014).

In particular, in process systems, improvements in quality, throughput, safety or constraint satisfaction are usually linked to more sophisticated and complex control and decision-making systems (Martí, 2015). However, independently of this, decisions have to be made at different time scales, like planning and scheduling the production in a time span of weeks to days, optimize the operation and the use of materials in a days to hours time-scale and finally, in an hour to minutes span, it is necessary to control and supervise the process considering current disturbances. This proposed point of view makes possible to compare the decision-making process to a hierarchically

organized one, where the higher layers process wider global information compared to the lower ones.

The existence of this different time scales in the decision-making process makes possible the fact that the outcomes of the upper layers can be directly related to the degrees of freedom of the lower or subordinated ones (Navia, 2012) defining their tasks and goals.

In Optimization, economic-based parameters are generally addressed in the objective function and traditionally economic optimization (linked to chemical process control) is carried in a multilayer hierarchical structure like the described above and is showed in Figure 1, where specifically the higher layer is called Real Time Optimization (RTO). This layer generally optimizes operating profits versus costs by computing optimal processes set points; these set points will be used in the supervisory control layer. In this layer advanced control algorithms are used in order to integrate information about process constraints and possible couplings between process variables or units. It is common the use of dynamic models while trying to formulate the proper optimization problem. Whereas the problem is dynamic, the resolution leads to find an optimal input trajectory of the decision variables over a finite time horizon. This can consider multiple inputs and multiple outputs -or MIMO- unlike the next layer (regulatory), which works with single input single outputs control loops mostly like proportional derivative integral -or PID- controllers. This architecture presents intuitively the separation by time scales presented before.

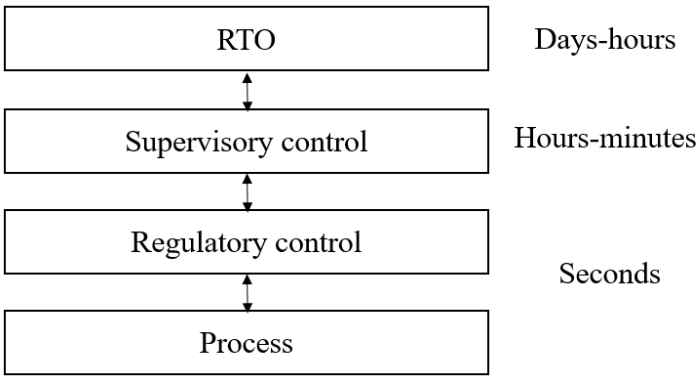

Figure 1: Traditional architecture and its time updating policy employed in process optimization and control in chemical process industries.

Considering the advances in design and operation of industrial plants and applying a hierarchical or layered regulatory system, the resulting process control problems and its respective optimization

problem will involve multiple units and are called large-scale process control. In this context, the term large-scale system is used to describe the plant, which includes a number of processing units linked to each other as a result of shared or interacting variables (Martí, 2015).

Perform optimal control in an industry with a large-scale process is a challenging task, because of the topology of the optimization problem which need to integrate information about the dependency that exists between subunits into the plant, also, handling this kind of problems allows to include more detailed models, which are harder to handle, but leads to more realistic solutions.

In order to enable next-generation operation in control and optimization in real industries, new methodologies capable of handling dynamic optimization process operation have to be proposed. The benefits of such work may be transformative to process operations in a new era of dynamic process operations (Ellis et al., 2014).

There are many industries in which the problems are or are transforming into large-scale systems. This document deals with two historical problems: (1) shared resources and distribution and (2) stochastic optimization. Considering this, different ways to solve them are proposed and compared: (1) centralized, when the problem is solved using its original topology. Here, two methods are studied: a usual optimization protocol and a projection-based technique. The second group of methods are (2) coordinated, where an extra layer is inserted in the hierarchical structure and the original problem's topology is modified. The coordinated methods studied are clipping and augmented Lagrangian. Finally, an experimental system is proposed to verify the performance of the different solution methods, using them in a control regime with a mobile horizon.

1.1. Motivation

Considering that: (i) advances in design and modeling of chemical processes means more and more complex dynamic models available, (ii) chemical plants consider even in their earlier design stages the interconnection of process streams as a way to optimize some type of resource, and (iii) economic, environmental and safety regulations tend to increase the number of restrictions on the operation of industrial processes, the number of problems currently considered as large-scale ones are increasing, so the study of strategies for their proper resolution and implementation are a critical point for the eventual use of these protocols in the control of industrial processes.

1.2. Objectives

The general objective of this work is to test a control methodology with a receding horizon that effectively works in an experimental system. This experimental environment emulates an industrial facility which has a large-scale system.

The specific objectives are presented below:

- Study the topology of historical large-scale problems.
- Propose solution methodologies for the studied problems, such as a coordinated based solution algorithm and the projection-based algorithm and study its convergence criterions.
- Test the performance of the methods in an experimental system and compare them with the current solving method approach.

2. Context

The explosive growth in technology in recent decades has produced more complex, integrated and interconnected industrial plants than their predecessors (Scattolini, 2009). The goal of the optimization of an industrial plant is generally to make the best possible decision considering a defined criterion; therefore, under the concept of optimization, the integration of these networks (e.g., mass, energy or others) is usually an additional complication because more information must be considered simultaneously to make the right decision. Such systems can be difficult or even impossible to manage and control if appropriate measures are not taken when designing the control strategies (Shahidi et al., 2015).

How a decision is made depends on the type of behavior of the system. This work discusses control systems with a hierarchical structure, in which decision-making is made in levels: lower levels make quick decisions, and higher levels make general decisions, which take longer amounts of time and consider more decision variables. These levels are connected through controlled variables; for example, the upper layers compute the set points to be implemented in the lower levels (Skogestad, 2000).

The decisions to be made in a plant with a hierarchical structure, in which the layered structure can be clearly seen, including the following:

1. Planning: This is related to what to produce and how is it produced.
2. Programming: In this stage, we consider the information examined in the planning layer and decide when to produce.
3. Optimization: This is done based on the programming. It happens in real time and uses feedback from specific points in the process.
4. Restrictive/supervisory control: Through dynamic control of the plant, this stage tries to increase the optimization capacity to control in real time.
5. Distributed/regulatory control: Through the collection of information about the states of the units, regulatory actions are implemented.

The operation of a given process can be carried out in two levels: RTO (Real-Time Optimization) and MPC (Model Predictive Control), which are shown in Figure 2. The upper layer is dedicated to improve the operation of the plant using a criterion (generally economic), whereas the lower level implements these decisions, maintains adequate performance and tries to comply with the process constraints (Ellis et al., 2014).

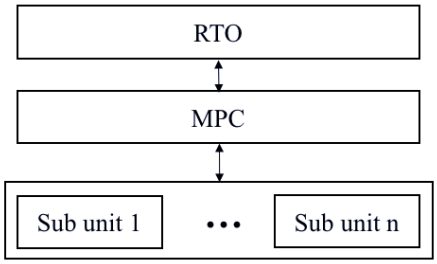

Figure 2: Hierarchical control in a two-layer decision scheme.

The current approach is an alternative to two-level control: the application of EMPC (Economic Model Predictive Control) is shown in Figure 3, which can react considering all of the degrees of freedom of the system collectively. This approach is expected to have a better performance than the two-level approach because some degrees of freedom can only be considered when working with a dynamic model. It is intuitive to expect that if more degrees of freedom are available, there is a better chance of managing the system in a proper or optimal manner. Another possible cause of the better performance is that the steady-state assumed model that is used in the RTO layer is not necessarily the same as that in the EMPC (Câmara et al., 2016), so different results could be reached by each layer.

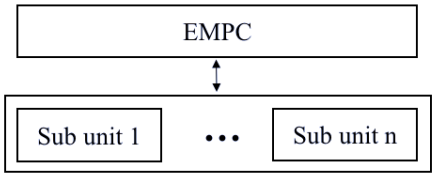

Figure 3: Proposed control scheme.

The main problem is that EMPC cannot currently be successfully solved in real time for large systems or those composed of many sub-units due to the common computational power, which is not able to solve a large number of simultaneous dynamic optimizations (Engell, 2007).

Because several layers of decision-making must work in real time, it is essential that the optimization and control levels be able to solve large-scale problems, which are defined as "problems that consider a large amount of information, which currently cannot be solved on a proper timescale".

Generally, large-scale systems are addressed by trying to decompose the problem into a series of smaller and more manageable problems to break up the central scenario, which considers all of the variables simultaneously, to a decentralized scenario by modifying its topology. This causes each section of the control layer to solve a small part of the original problem. The decentralized approach is the most popular method in the industry because of its simplicity of design and maintenance (Kano and Ogawa, 2010). Figure 4 shows a two-level hierarchical control in a decentralized approach, which can cause two possible errors: it can generate a sub-optimal operation because the control action found as a solution does not consider a conjugated scenario, and the control action found as a solution may not be feasible to apply (Martí, 2015).

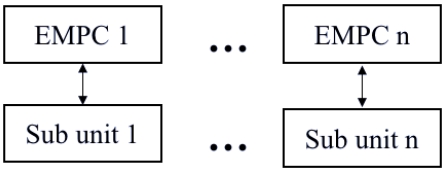

Figure 4: Hierarchical two-layer control in a decentralized approach.

To be able to effectively solve and implement the regulation of a large-scale system, a structure similar to that in Figure 5 is proposed, in which a coordination layer handles the information delivered by each EMPC in a decentralized manner and drives the system to a certain point while trying to follow an optimal trajectory. Recent studies and applications of this methodology can be found in Cheng et al. (2007), Lucia et al. (2013), and Shahidi et al. (2015).

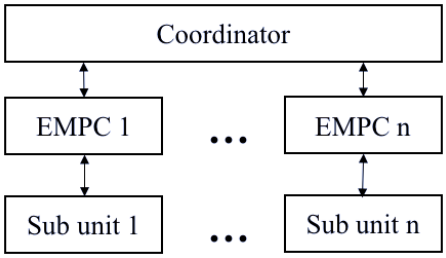

Figure 5: Coordinated EMPC approach.

Taking this into account, this study aims to implement and test resolution methodologies for large-scale problems in an experimental system, which can occur when: (1) facing a problem with shared limited resources, and (2) calculating the optimal trajectory of a process when one or more variables have uncertainty, and the problem is solved using a two-stage programming methodology.

The main contributions of this work are (1) the use of a projection-based method as an alternative to solve EMPC in a large-scale problem for resource distribution systems and for stochastic optimization and (2) an experimental system in which the implementation of the optimization protocols can be tested and compared.

3. Problem definition

Several historical examples have led to the formulation of large-scale problems, which have motivated the search for solution methods. Among them, two are reviewed here, and an experimental implementation is proposed for each of them in later sections.

3.1. Shared resources and distribution

The main characteristics of these kinds of systems and the reason why they are considered large-scale problems at an industrial level are: they are composed by a large number of sub-units, in which a limited amount of resources is used together, and some sub-units can produce them, whereas others can consume them. Systems with these characteristics have been widely explored in the past (Findeisen et al., 1980).

When a system can be described as n sub-units with N shared resources (Figure 6), the optimization problem can be written as Equation (1).

$$P_c^1: \quad \min_{[\boldsymbol{\mu},\boldsymbol{\nu}]} J(\boldsymbol{\mu},\boldsymbol{\nu}) \tag{1a}$$

s.t.

$$\boldsymbol{f_i}(\dot{\boldsymbol{x}}_{\boldsymbol{i}}, \boldsymbol{x_i}, \boldsymbol{\nu_i}, \mu_{i1}, \dots, \mu_{iN}, t) = 0, \qquad \forall i = 1, \dots, n, \ \ \forall t \in [0, T] \tag{1b}$$

$$\boldsymbol{g_i}(\dot{\boldsymbol{x}}_{\boldsymbol{i}}, \boldsymbol{x_i}, \boldsymbol{\nu_i}, \mu_{i1}, \dots, \mu_{iN}, t) \leq 0, \qquad \forall i = 1, \dots, n \quad \forall t \in [0, T] \tag{1c}$$

$$r_j\left(\left(\mu_{ij}\right)_{i=1}^{n}\right)_j \coloneqq \sum_{i=1}^{n} R_{ij}(\mu_{ij}) \leq R_{T\,j} \qquad \forall j = 1, \dots, N \quad \forall t \in [0, T] \tag{1d}$$

where $[\boldsymbol{\mu}, \boldsymbol{\nu}]$ denotes the control action that optimizes the full system under the $J(\boldsymbol{\mu}, \boldsymbol{\nu})$ criterion. Specifically, $\boldsymbol{\mu} = \left(\left(\mu_{ij}\right)_{i=1}^{n}\right)_{j=1}^{N}$ is the trajectory for the decision variables, which contains information about the N shared resources (potentially consumed by more than one unit in the same time span), and $\boldsymbol{\nu}$ contains information about the local resources or those that can be consumed in a specific unit (i.e., resources that are not shared) due to its nature, the nature of the sub-unit, or due to a process constraint. Hence, calculate $\boldsymbol{\mu}$, is to compute the decision of what mixture of resources will be used in each unit for each time interval, when considering a mobile horizon with an economic objective function

In addition, $\boldsymbol{f_i}$ and $\boldsymbol{g_i}$ denote the models and the constraints associated with sub-system i, respectively. If the time $t \in [0, T]$, then $\mu_{ij}: [0, T] \to \mathbb{R}$ is the trajectory of the decision variable of resource j applied to subunit i.

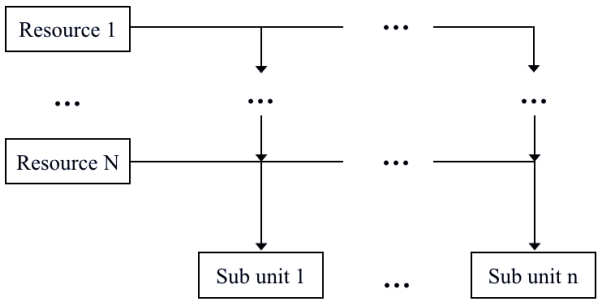

Figure 6: Diagram of the proposed problem, which is a system with $\boldsymbol{N}$ resources that feeds $\boldsymbol{n}$ subunits.

In the global restriction (1d), the parameter $R_{T\,j}$ refers to the availability of resource j. The constraint (1d) deserves special attention because it prevents the problem from being solved for each sub-unit separately (or decentralized) and transforms the problem P_C^1 into a large-scale

problem. In particular, this constraint suggests that the total amount of the specific resource j being used in all of the sub-units simultaneously, which is represented by $r_j\left(\left(\mu_{ij}\right)_{i=1}^{n}\right)_j$, has a maximum allowable quantity. Similarly, $R_{ij}(\mu_{ij})$ denotes the quantity of resource j being consumed specifically in sub-unit i.

The time required to solve a problem with a topology similar to P_C^1 can compromise the application in real time for a system operated under an optimal control regime, and another approach must be found to solve the problem effectively.

3.2. Stochastic optimization

A stochastic problem is generally an optimization problem in which a set of variables that affect the behavior of the process is not known, but statistical information about them is available. An example is the calculation of an optimal production policy when the quality of the raw materials is unknown, but historical data can be used to characterize uncertain information (Navia et al., 2014). The uncertain variable set is called a stochastic variable, which is denoted by $\boldsymbol{\xi}$.

If the scalar objective function J is the optimization criteria, the problem can be represented as in Equation (2), where $\boldsymbol{\mu}$ is the decision control variable, $\boldsymbol{x}$ the states of the system, J represents the optimization criteria, and $\boldsymbol{f}$ and $\boldsymbol{g}$ denote the models and constraints associated with the system, respectively. The direct resolution of P_C^2 yields an optimal control action, which is a function of the value of the stochastic variable; i.e., $\boldsymbol{\mu}^*(\boldsymbol{\xi})$. The problem is that the real values of the stochastic variables are unknown; therefore, the applicable control action cannot be determined.

$$\min_{\boldsymbol{\mu}} J(\boldsymbol{\xi}, \boldsymbol{\mu}, \boldsymbol{x}) \quad (2a)$$

s.t.

P_c^2:

$$\boldsymbol{g}(\boldsymbol{\xi}, \boldsymbol{\mu}, \boldsymbol{x}) \leq 0 \quad (2b)$$

$$\boldsymbol{f}(\boldsymbol{\xi}, \boldsymbol{\mu}, \boldsymbol{x}) = 0 \quad (2c)$$

$$\boldsymbol{\xi} \in \Xi,\ \boldsymbol{\xi} \sim PDF\left(\boldsymbol{\eta}_{\xi}, \boldsymbol{\Sigma}_{\xi}\right) \quad (2d)$$

In (2) $\boldsymbol{\xi}$ can be described with a probability density function PDF with a mean $\boldsymbol{\eta}_{\xi}$ and covariance matrix $\boldsymbol{\Sigma}_{\xi}$.

Several methods can be used to solve (2) and to determine a particular control action, such as robust optimization. Stochastic optimization relies on ideas from robust control and also includes information obtained from the historical behavior of the stochastic variable, and it uses a two-stage method.

Two-stage programming is a particular case of the multi-stage programming methodology, which was introduced in the 1950s (Dantzig, 1955) to solve large-scale linear optimization problems. It is conceptualized by the premise that there are several stages of knowledge of the variables, and these variables can be defined and computed in *decision stages*. In this case, the two proposed stages are (Birge and Louveaux, 1997):

1. Here-and-now (first stage variables): these are related to moving the state of the system from one point to another while only knowing the statistical information about the stochastic variables and looking for feasibility.
2. Wait-and-see (second stage variables): these are related to trying to address any mistakes made in the first stage, assuming that there is a procedure that allows the real value of the stochastic variables to be determined, and then proposing a corrective action (searching for optimality).

The two-stage programming methodology has been widely discussed and proven in economic control approaches by other authors (Lucia et al., 2013; Martí et al., 2015).

A modified version of P_c^2 is shown in Equation (3), which is reformulated following a two-stage approach for a discretized PDF of the stochastic variables. In this case, n denotes the number of discretized scenarios i from the original PDF shown in (2d), each of which has a probability Π_i $i \in \{1,2,\dots,n\}$, and N represents the number of resources, as in the previous example. In this reformulation, t_1 represents the limit between the first and second decision stage, and T represents the full control horizon. The control trajectory is divided as $\boldsymbol{\mu} = [\boldsymbol{\mu^0}, \boldsymbol{\mu^1}]^T$, where $\boldsymbol{\mu^0} = \left[\boldsymbol{\mu_1^0}, \dots, \boldsymbol{\mu_n^0}\right]^{\boldsymbol{T}}$ stands for the first stage variables, $\boldsymbol{\mu^1} = \left[\boldsymbol{\mu_1^1}, \dots, \boldsymbol{\mu_n^1}\right]^{\boldsymbol{T}}$ represents the second stage variables, $\boldsymbol{\mu_i^k} = \left[\mu_{i1}^k, \dots, \mu_{iN}^k\right]$ represents the decision variables for stage k in scenario i, and μ_{ij}^k represents the trajectory of resource j in scenario i for decision stage k.

$$P'^{2}_{C}: \quad \min_{\boldsymbol{\mu}} \sum_i \Pi_i J_i(\boldsymbol{\mu_i^0}, \boldsymbol{\mu_i^1}, \boldsymbol{\xi}_i) \tag{3a}$$

s.t.

$$\boldsymbol{f}\left(\boldsymbol{x_i}, \dot{\boldsymbol{x}}_{\boldsymbol{\iota}}, \boldsymbol{\mu_i^0}, \boldsymbol{\xi_i}\right) = \mathbf{0}, \quad \boldsymbol{x_i}(t=0) = \boldsymbol{x^0}, \qquad t \in [0, t_1), \tag{3b}$$

$$\boldsymbol{f}\left(\boldsymbol{x_i}, \dot{\boldsymbol{x}}_{\boldsymbol{\iota}}, \boldsymbol{\mu_i^1}, \boldsymbol{\xi_i}\right) = \mathbf{0}, \quad \boldsymbol{x_i}(t_1) = \boldsymbol{x_i}(t_1, \boldsymbol{\xi_i}), \quad t \in [t_1, T), \tag{3c}$$

$$\boldsymbol{\mu_i^0} = \boldsymbol{\mu_{i+1}^0}, \qquad i = 1, \dots, n-1 \tag{3d}$$

$$r_j\left(\mu_{ij}^k\right) \leq R_{T_j}, i = 1, \dots, n; \; j = 1, \dots, N, k = 0{,}1; t \in [0, \mathrm{T}] \tag{3e}$$

Constraint (3d) refers to the first decision stage, and it is called a non-anticipativity constraint. This equality constraint is imposed to find a control action that, although not optimal in the usual sense, covers the greatest number of problems that result from not knowing the value of the stochastic variable during the first stage in a concise form. Furthermore, it represents the coupling between scenarios; in this case, it prevents the problem from being solved separately for each probabilistic scenario.

Shared resources and stochastic optimization problems have a similar structure. They have a decentralized structure with the exception of one constraint in particular: the constraint referred to as the availability of resources in the shared resources problem and as the non-anticipativity constraint in the stochastic optimization problem.

Taking this into consideration, although these restrictions are somewhat different depending on the problem, a coordination layer can be proposed for the resolution of these two different problems. Furthermore, the methodology will be contrasted with others, such as the use of projections, to compare them in a benchmark experimental system.

4. Experimental Implementation

4.1. Experimental System

The system is intended to prove and compare the performances of different resolution methods for the shared resources problem (P_C^1) and the stochastic optimization problem that are reformulated with the two-stage approach ($P_C'^2$). They are demonstrated in an experimental setup composed by a four-unit benchmark feed with fresh water as shown in Figure 7.

The use of this particular experimental system was proposed because it can easily emulate the behavior of a shared resources problem by using $n > 1$ tanks an $N > 1$ resources and also the stochastic optimization problem by using $n = 1$ tanks and $N \geq 1$ resources while making a minimum amount of adjustments between problems and using inexpensive instrumentation.

The control task is to track the time-varying reference temperatures to drive the state of the system to consecutive and different steady-state temperatures using two different resources with an economic criterion. The cross-sectional area of the units is $A = 258.5\ cm^2$, and the level of the water inside every unit is assumed to be constant, which results in a volume $V = 2\ L$ in each unit.

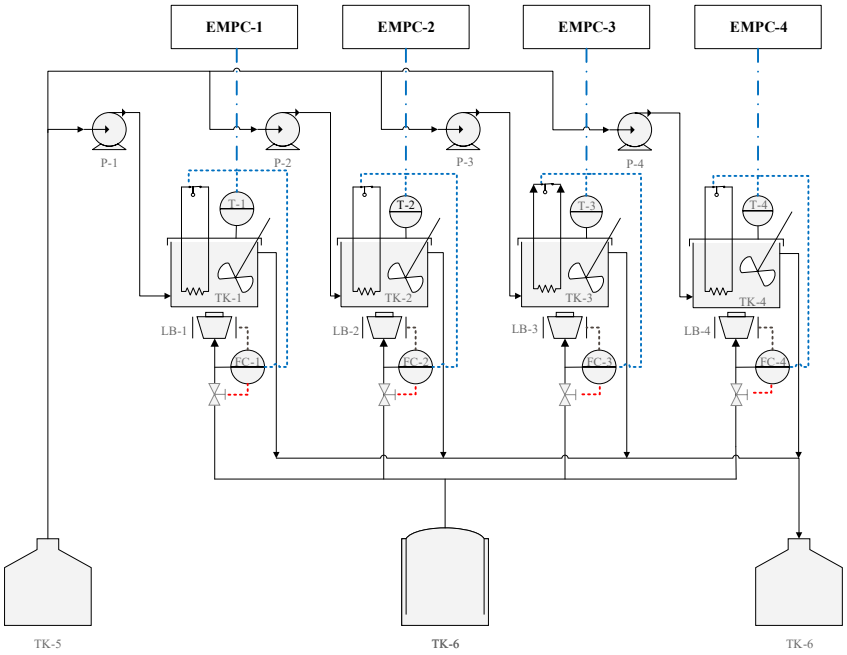

Figure 7: Schematic diagram of the proposed system.

Fresh water is fed using peristaltic pumps (Masterflex 07528-10, head 77800-52), which are labeled P-1 to P-4 in Figure 7, and each unit (TK-1 to TK-4) has a mechanical agitator (Boeco OSD-20 S65). The first resource is a resistance (helical 2000W/220V) connected to a solid-state relay (ANV SSR-25LA). The second resource is a Bunsen burner (LB-1 to LB-4) connected to a mass gas flowmeter (Cole-Parmer 32907-69), labeled FC-1 to FC-4 in Figure 7, which is connected in a serial manner with a set point unit (Cole-Parmer 32907-85). In this system, the gas is ignited using a spark plug (GASFIX IN: 3VDC). Labels T-1 to T-4 in Figure 7 refer to the thermocouples (VT-DKSGD-100L-1). The control layer is implemented using a FATEK FBs-20MAR-2AC PLC, which is connected to five additional units (FBs 6TC, FBs 6AS, and three FBs 4DA). The computer has an Intel(R) Core™ i7-4770 processor with a CPU @ 3.40 GHz and 8.00 GB of RAM.

4.2. Model of the system

The dynamics of the proposed system can be described by a set of differential equations as shown in Equation (4), where ρ and C_p are the density and calorific capacity of water, respectively, q is the water flow, T_i represents the temperature in unit i, T_a is room temperature, T_{in} is the inlet water temperature and μ_{i1} and μ_{i2} describe the trajectories of heating resources one and two in unit i. The original model considers the dependence of the water's heat capacity and the heat transfer coefficient (h_c) as a function of the state; nevertheless, because the economic control problem is implemented in a receding horizon scheme, successive linearization is performed.

$$\frac{\partial T_i}{\partial t} = \frac{q\rho C_p(T_i)(T_{in} - T_i) - h_c(T_i, T_a)A(T_i - T_a) + \mu_{i1} + \mu_{i2}}{\rho V C_p(T_i)} \ , i = 1,2,3,4 \qquad (4)$$

As described above, a receding control horizon is considered during the formulation and further application of the problems. A time span (τ) is proposed, and the optimization problems are solved using control vector parametrization by dividing the continuous trajectory into Θ individual decisions. Both τ and Θ are tuned for each problem to adapt to the solution times, which are different for each solution method and problem.

4.3. Shared resources

The problem P_C^1 is reformulated using control vector parametrization with $\tau = 180\ s$ and $\Theta = 16$, and because every continuous trajectory (μ_{ij}) shown before is now a vector, as shown in Equation (5), the problem has 128 decision variables, where $\boldsymbol{T_i}$ and $\boldsymbol{T_{sp}} \in \mathbb{R}^{\Theta}$, and $\boldsymbol{P} = [\boldsymbol{P_1}, \boldsymbol{P_2}]^{\boldsymbol{T}}$ is the price vector. In this problem, $\boldsymbol{P_1} = 1 \times 10^2 \cdot \boldsymbol{e}$ and $\boldsymbol{P_2} = 2 \times 10^3 \cdot \boldsymbol{e}$ for each resource, where $\boldsymbol{e} = [1,1,\dots,1] \in \mathbb{R}^{\Theta}$. The parameterized decision variable can be shown as $\boldsymbol{\mu} = [\boldsymbol{\mu_{11}}, \boldsymbol{\mu_{12}}, \dots \boldsymbol{\mu_{41}}, \boldsymbol{\mu_{42}}]^T$, where $\boldsymbol{\mu_{ij}} \in \mathbb{R}^{\Theta}\ \forall i = 1 \dots 4, \forall j = 1, 2$.

$$\min_{\boldsymbol{\mu}} \sum_{i=1}^{4} \left(\left\| \boldsymbol{T_i} - \boldsymbol{T_i^{sp}} \right\|^2 + \boldsymbol{P} \cdot [\boldsymbol{\mu_{i1}}, \boldsymbol{\mu_{i2}}]^T \right) \qquad (5a)$$

s.t.

$$0 \leq \sum_{i=1}^{4} \boldsymbol{\mu_{ij}}(l) \leq \mu_j^{max} \qquad l = 1, \dots, \Theta;\ \forall \mathrm{j} \qquad (5b)$$

In this case, the units are fed with $q = [1.07, 1.00, 0.56, 0.82]\ L/min$ of water with an inlet temperature of $T_{in} = 22$℃. The first resource is a resistance with a maximum power of $\mu_1^{max} = 5\ kW$. The second resource is a Bunsen burner, which can provide a maximum power of $\mu_2^{max} = 0.7\ kW$. The initial temperature in each unit is initially set to $T_0 = [26.3, 25.9, 27.7, 25.4]$ ℃. Finally, room temperature is set to $T_a = 25$ ℃, and a convection factor of $h_c = 0.100\ kW/m^2$℃ is assumed.

The control task is to track the time-varying temperature references to drive the state of the system into three consecutive and different steady-state temperatures using two different resources in each unit with an economic criterion. The overall duration of the control task is set to $1050\ s$ and is divided into three consecutive intervals of $400\ s$, $400\ s$, and $250\ s$. The set points are $T^{sp} = [35, 31, 33, 30]$ ℃ for the first time span, $T^{sp} = [29, 36, 28, 34]$ ℃ for the second time span and $T^{sp} = [33, 33, 31, 32]$ ℃ for the last time span.

4.4. Stochastic optimization

In this situation, only unit Tk-1 will be used. The problem $P_C'^2$ reformulated using control vector parametrization with $\tau = 180\ s$ and $\Theta = 11$ is shown in Equation (6), which results in a problem

with 154 decision variables. This unit is fed with $q = [1]\ L/min$ of water with an inlet temperature that follows the assumed distribution shown in Figure 8.

$$\min_{\boldsymbol{\mu}} \sum_{i=1}^{7} \Pi_i \left(\left\| \boldsymbol{T_1^i} - \boldsymbol{T^{sp}} \right\|^2 + \left(\boldsymbol{P_1^T} \cdot \left[\boldsymbol{\mu_{i1}^0}, \boldsymbol{\mu_{i1}^1} \right]^{\mathrm{T}} + \boldsymbol{P_2^T} \cdot \left[\boldsymbol{\mu_{i2}^0}, \boldsymbol{\mu_{i2}^1} \right]^{T} \right) \right) \qquad (6a)$$

s.t.

$$\boldsymbol{\mu_{ij}^0} = \boldsymbol{\mu_{i+1,j}^0}, \qquad i = 1,2, \dots ,6;\ \forall j \qquad (6b)$$

$$0 \leq \boldsymbol{\mu_{ij}}(k) \leq \mu_j^{max}\ , \qquad k = 1,2, \dots ,11;\ \forall j \qquad (6c)$$

where $\boldsymbol{T_1^i}$ and $\boldsymbol{T^{sp}} \in R^{\Theta}$, and $\boldsymbol{P}$ is the price vector with appropriate dimensions and magnitudes $P_1 = 1 \times 10^2$ and $P_2 = 2 \times 10^3$. The parameterized decision variable can be shown as $\boldsymbol{\mu} = [\boldsymbol{\mu_{11}}, \boldsymbol{\mu_{12}} \dots \boldsymbol{\mu_{71}}, \boldsymbol{\mu_{72}}]^T$, where $\boldsymbol{\mu_{ij}} \in \mathbb{R}^{\Theta}, \forall i = 1 \dots 7, \forall j = 1,2$.

The first resource is a heating element with a maximum power of $\mu_1^{max} = [2]\ kW$. The second resource is a Bunsen burner, which can provide a maximum power of $\mu_2^{max} = [0.3]\ kW$. The initial temperature of the unit is set to $T_0 = [22]$ ℃. Finally, room temperature is set to $T_a = [25]$ ℃, and a convection factor of $h_c = [0.100]\ kW/m^2$℃ is assumed.

The temperature of the inlet water is the stochastic variable and it is assumed to be unknown, but it follows the PDF shown in Figure 8. The overall duration of the control task is set to 900 s, and it is divided into three consecutive intervals of 300 s. The set points for the time spans are $T^{sp} = [35]$ ℃, $T^{sp} = [27]$ ℃, and $T^{sp} = [32]$ ℃ , respectively.

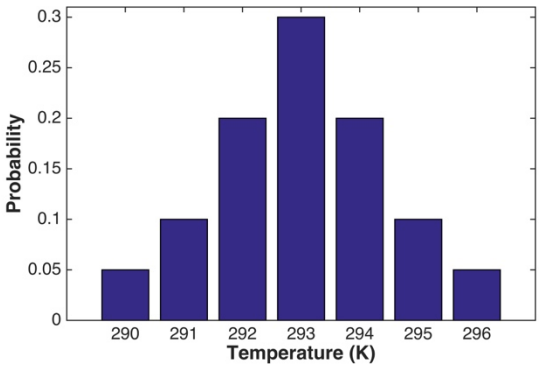

Figure 8: Proposed distribution of the stochastic variable.

5. Solution methodology

A general formulation of the equations (5) and (6) is presented by Equation (7):

$$\min_{\boldsymbol{\mu}=(\boldsymbol{\mu_i})_{i=1}^{n}} \sum_i F_i(\boldsymbol{\mu_i}) \qquad (7a)$$

$$\boldsymbol{A\mu} \leq \boldsymbol{b} \qquad (7b)$$

$$\boldsymbol{C\mu} = \boldsymbol{d} \qquad (7c)$$

Where F represents for the objective function corresponding to one process unit or to one discrete scenario in the space of probabilities, and can be understood as $F_i\ (\boldsymbol{\mu_i}) = J\ (\boldsymbol{x}\ (\boldsymbol{\mu_i}), \boldsymbol{\mu_i})$ for $i = 1, \dots, 4$ in equation (5) or for $i = 1, \dots, 7$ in equation (6).

In order to represent the equality and inequality constraints in a general manner, $\boldsymbol{A}$, $\boldsymbol{b}$, $\boldsymbol{C}$ and $\boldsymbol{d}$ are introduced to the formulation. It must be noticed that, only when $\boldsymbol{C} = \boldsymbol{0}$, Equation (7) represents Equation (5), since there are no equality constraints in the definition of the shared resources problem.

Two methodologies will be used to solve the general large-scale problem described above. The first corresponds to a centralized approach, which solves the optimization problem through the use of projections. The second methodology, which is called augmented Lagrangian, uses a coordination layer to solve a modified dualized version of the original formulations. These methodologies take advantage of the structure of the problem, using in their favor the separable nature of the objective function and the topology of the linking constraints. The performance of these two methods will be compared with the solution computed with a regular optimization package.

A third methodology called clipping is also proposed, which is used only in shared resources problems to obtain a feasible point that is used as the initialization for the augmented Lagrangian methodology.

The proposed methodologies work in a similar manner: giving initial values to the decision variables the states of the system are calculated using the dynamic equations. This information is fed into the optimization routines in order to calculate the gradients, to evaluate the objective functions and to obtain the optimal points for each case according to the proposed methodology,

which are theoretically equal to those obtained by the traditional optimizer, but a shorter calculation time is expected. The algorithms are described below.

5.1. Centralized approach solving using projections

During the 1960s, a novel method for minimizing bounded problems with continuously differentiable objective functions was proposed (Goldstein, 1965, 1964) by using consecutively gradient steps and projections to search for optimality and feasibility, respectively. The projection operator is defined in Equation 8, where $P_A\boldsymbol{\mu}$ is the projection of the point $\boldsymbol{\mu}$ over the set A; in this context, the set A represents the feasible points and it is directly related to the constraints. The operator works by taking a point $\boldsymbol{\mu}$ and "projecting it" into the set A: if $\boldsymbol{\mu} \in A$, then $P_A\boldsymbol{\mu} = \boldsymbol{\mu}$, but if $\boldsymbol{\mu} \notin A$, then $P_A\boldsymbol{\mu}$ will be the closest point in A to $\boldsymbol{\mu}$, which always exists and is unique when A is nonempty, convex and closed (Hiriart-Urruty, 1993).

$$P_A\boldsymbol{\mu} \coloneqq \arg\min_{\tilde{\boldsymbol{\mu}} \in \boldsymbol{A}} \|\boldsymbol{\mu} - \tilde{\boldsymbol{\mu}}\| \qquad (8)$$

where $\tilde{\boldsymbol{\mu}}$ represents the projected decision variable. To be able to evaluate the projection instead of solving the optimization problem shown in Equation (8), it can be proven that if $A = \ker(\boldsymbol{R})$, then $P_A\boldsymbol{\mu}$ is defined as in Equation (9) when $\boldsymbol{R} \cdot \boldsymbol{R}'$ is invertible (Combettes, 2017).

$$P_A\boldsymbol{\mu} = \boldsymbol{\mu} - \boldsymbol{R}' \cdot (\boldsymbol{R} \cdot \boldsymbol{R}')^{-1} \cdot \boldsymbol{R} \cdot \boldsymbol{\mu} \qquad (9)$$

As mentioned previously, it is necessary the equality constraints to be linear since the constraints must form a convex set (thus, the projection operator keeps its geometric interpretation). The inequality constraint may be nonlinear while they comply a convex nature.

The proposed projection-based methodology used for solving a large-scale centralized problem is described in Algorithm 1 (Beck and Teboulle, 2009) and it is based in alternated gradient steps and projections, while the initialization point is transformed into the optimal one.

ALGORITHM 1

1. Set $k = 0$, $t_0 = 1$ and then propose and initialize $\boldsymbol{\mu_0}$ and ε.
2. Set $\bar{\boldsymbol{\mu}}_{\boldsymbol{k}} = \boldsymbol{\mu_0}$.
3. Compute consecutively:

a. $\boldsymbol{\mu_{k+1}} = P_A\left(\overline{\boldsymbol{\mu}}_{\boldsymbol{k}} - \frac{1}{\mathcal{L}(F)} \cdot \nabla_{\boldsymbol{\mu}} \mathrm{F}(\overline{\boldsymbol{\mu}}_{\boldsymbol{k}}) \right)$

b. $t_{k+1} = \frac{1+\sqrt{1+4 \cdot t_k^2}}{2}$

c. $\overline{\boldsymbol{\mu}}_{\boldsymbol{k+1}} = \overline{\boldsymbol{\mu}}_{\boldsymbol{k}} + \frac{t_k - 1}{t_{k+1}} (\boldsymbol{\mu_{k+1}} - \boldsymbol{\mu_0})$

4. Compare $\overline{\boldsymbol{\mu}}_{\boldsymbol{k}}$ and $\overline{\boldsymbol{\mu}}_{\boldsymbol{k+1}}$ over the tolerance $\boldsymbol{\varepsilon}$. If $\| \overline{\boldsymbol{\mu}}_{\boldsymbol{k+1}} - \overline{\boldsymbol{\mu}}_{\boldsymbol{k}} \|_2 < \varepsilon$, stop; otherwise, set $k + 1$ and go back to step 3.

In step 3.a., $\mathcal{L}(F)$ represents the Lipschitz constant of $\nabla_{\boldsymbol{\mu}}\mathrm{F}$ (Beck and Teboulle, 2009).

Application to shared resources and distribution problems: In this case, we define the set K, which represents the points in $\mathbb{R}^{N \cdot n \cdot \Theta}$ that fulfill the resource availability constraint (5b). Therefore, Algorithm 1 must be computed using $A = K$ in step 3.a. Figure 9(a) shows an example of how the projection operator works in $\mathbb{R}^2$; i.e., a system with two units sharing one limited resource, where the orange dotted lines shows the trajectories of different points in $\mathbb{R}^2$ being "projected" over the set K by leaving it over the nearest feasible point as described in the projection definition. A generalization must be made to project a point in higher dimensional constraints.

Application to stochastic optimization: In this case, we define the set V, which represent the points that fulfill the constraint (6b), K, which fulfills the constraint (6c), and finally a third set $C = V \cap K$.

Note that the nature of the set K, which is linked to constraint (6c), is similar to constraint (5b) in the previous problem because both refer to the total resource availability; therefore, the projection calculation can be handled in a similar manner. Figure 9(b) shows an example of how the projection operator works in $\mathbb{R}^2$; i.e., a system with a stochastic variable with two discretized scenarios that are using one limited resource. In Figure 9(b), set V corresponds to the dotted blue line that represents the non-anticipativity constraint, and set C corresponds to the green line inside set K. The orange dotted lines show the trajectories of different points in $\mathbb{R}^2$ being "projected" over set C by leaving it over the nearest feasible point as described in the projection definition. A generalization must be made to project a point in higher dimensional constraints.

Because the solution of the stochastic optimization requires the projection of the decision variable over the intersection of two sets, it is necessary to define an alternative procedure to Algorithm 1 in step 3.a to change the projection from P_A to $P_{C=V\cap K}$, where the projection $P_C\boldsymbol{\mu}$ can be approximated using Algorithm 2. Algorithm 2 iteratively finds the projection of the point $\overline{\boldsymbol{\mu}}$ over the set C. The convergence of this method is studied in Briceño-Arias (2015).

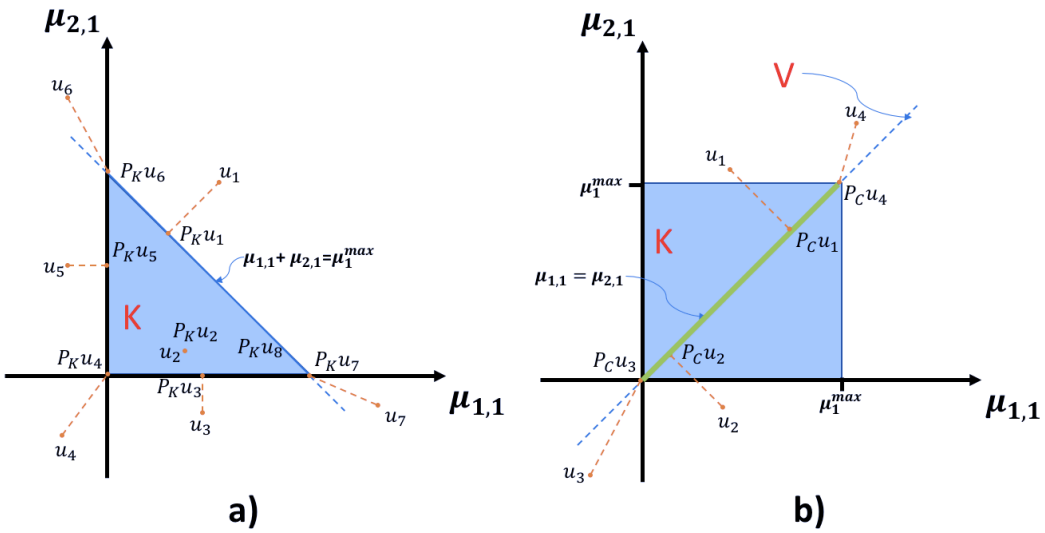

Figure 9: Graphical representation of the projections in $\mathbb{R}^2$. a) Examples of the projection of the point $\boldsymbol{u_i}$ over the set $\boldsymbol{K}$ (blue region), b) examples of the projection of the point $\boldsymbol{u_i}$ over the set $\boldsymbol{C = V \cap K}$. The set $\boldsymbol{C}$ is represented by the green solid line.

ALGORITHM 2

1. Set $k' = 0$ and propose and initialize $\boldsymbol{\mu_0}$ and $\gamma < 2$.
2. Compute $\boldsymbol{\mu_V} = P_V\overline{\boldsymbol{\mu}}$
3. Compute consecutively
    a. $\boldsymbol{x_{k'}} = P_V\boldsymbol{\mu_{k'}}$
    b. $\boldsymbol{s_{k'}} = (2-\gamma)\boldsymbol{x_{k'}} - \boldsymbol{\mu_{k'}} + \gamma\boldsymbol{\mu_V}$
    c. $\boldsymbol{p_{k'}} = P_K\boldsymbol{s_{k'}}$
    d. $\boldsymbol{\mu_{k+1}} = \boldsymbol{\mu_k} + \boldsymbol{p_k} - \boldsymbol{x_k}$
4. Compare $\boldsymbol{\mu_k}$ and $\boldsymbol{\mu_{k+1}}$ over a tolerance. If necessary, set $k = k + 1$, and go back to step 3. If the convergence criterion is reached, point $\boldsymbol{\mu_k}$ represents the projection of the initial point $\overline{\boldsymbol{\mu}}$ over both sets V and K.

5.2. Coordination strategies

Augmented Lagrangian: This methodology is based on the combination of two different approaches: the Lagrange function and the penalty method. It consists of removing the constraints and "augmenting" the objective function. If $\boldsymbol{u}$ represents the Lagrange multipliers of the problem, the augmented Lagrangian can be defined as:

$$L(\boldsymbol{\mu}, \boldsymbol{u}, \lambda) = \phi(\boldsymbol{\mu}) + \boldsymbol{u}^{\boldsymbol{T}} \boldsymbol{f}'(\boldsymbol{\mu}) + \frac{1}{2}\lambda \|\boldsymbol{f}'(\boldsymbol{\mu})\|^2 \qquad (10)$$

where ϕ represents the objective function, and $\boldsymbol{f}'$ represents the equality constraints of the problem. The augmented Lagrangian presented in Equation (10) is a proper formulation for problems with topologies like P'^2_C (by using $\boldsymbol{f}'$ as the non-anticipativity constraint (3d) or (6b)), but this formulation can be extended to the case in which both equality and inequality constraints are present in the original problem (Kall and Wallace, 2008), such as in the shared resources and distribution problem P^1_C. A generalized augmented Lagrangian function ($\tilde{L}$) suitable for problems with equality and inequality constraints is shown in Equation (11)

$$\tilde{L}(\boldsymbol{\mu}, \boldsymbol{u}, \lambda) = \phi(\boldsymbol{\mu}) + \frac{1}{2\lambda}\sum_{i=1}^{m}\{\{\max[0, \boldsymbol{u}(\boldsymbol{i}) + \lambda f_i''(\boldsymbol{\mu})]\}^2 - \boldsymbol{u}(\boldsymbol{i})^{\boldsymbol{2}}\} \quad (11)$$

where m is the total number of constraints in the original problem, and $\boldsymbol{u} \in \mathbb{R}^m$. In this case, and to use this formulation for the problem P^1_C, the vector $\boldsymbol{f}''$ is referred to as constraint (1d) or (5b), which is reformulated as an equality constraint using slack variables.

The intention of these two formulations is to control the parameters $\boldsymbol{u}$ and λ, and thus $\lambda \to \infty^+$, to eliminate infeasibilities, and $\boldsymbol{u} \to \widehat{\boldsymbol{u}}$ is the proper Lagrange multiplier vector. When these two conditions are reached, we can also guarantee that the decision variable has an optimal value. Note that when $\lambda = 0$, the generalized augmented Lagrangian is equal to the Lagrangian, and when $\boldsymbol{u} = \boldsymbol{0}$, the generalized augmented Lagrangian is transformed to the objective function with the penalty method.

At the beginning of most resolution processes, there is generally no knowledge about the magnitude of the proper multipliers, which can be a problem for the solution. Therefore, it is generally recommended to include an upper bound for $\boldsymbol{u}$.

The algorithm that will be used to solve a problem using the augmented Lagrangian method will be shown using the stochastic optimization problem as an example. Subsequently, the changes that must be made to use the algorithm with the shared resources problem will be presented.

To decentralize P_C^2, an additional parameter $\bar{\bar{\boldsymbol{\mu}}}$ is introduced that results in P_D^2, which has the appropriate topology to be treated with the augmented Lagrangian method.

$$P_D^2: \quad \min_{\boldsymbol{\mu}\in[\mu_{lb},\mu_{ub}]} J_i(\boldsymbol{\mu_i}, \Pi_i, \xi) \qquad \forall i \text{(12a)}$$

s.t.

$$\boldsymbol{\mu_i} = \bar{\bar{\boldsymbol{\mu}}}, \qquad i = 1, \dots, n \quad \text{(12b)}$$

$$t \in [0, \tau/\Theta]$$

Algorithm 3 describes the proposed procedure for solving a problem like P_D^2 when each scenario has a probability Π_i assigned.

ALGORITHM 3

1. Find $\boldsymbol{\mu^0}$ by solving P_D^2 while not considering the constraint (12b).
2. Set $k = 0$, $\lambda^k = 1$, and propose an initial Lagrangian multiplier vector $\boldsymbol{u^k} \in \mathbb{R}^m$.
3. Compute $\bar{\bar{\boldsymbol{\mu}}}^{\boldsymbol{k}} = \sum_i \boldsymbol{\mu_i^k} \cdot \Pi_i$.
4. Find $\boldsymbol{\mu^k} = [\boldsymbol{\mu_1^k}, \dots, \boldsymbol{\mu_n^k}]$ by solving the following bounded decentralized problem:

$$\min_{\boldsymbol{\mu_i^k}\in[\mu_{lb},\mu_{ub}]} L_i\left(\boldsymbol{\mu_i}, \boldsymbol{u_i^k}, \lambda^k\right)$$

where i represents an individual scenario, and $\boldsymbol{\mu_i^k}$ represents the decision variables for the scenario i in iteration k. In this case, L is defined as in Equation (10), with the dualized constraint from Equation (12b).

5. Check constraint (12b) $\forall i$ over a tolerance. If true, $\boldsymbol{\mu^k}$ corresponds to an optimal value, and $\boldsymbol{u^k}$ is the proper multiplier vector. If false, move to step 6.
6. Compute consecutively:
   a. $\lambda^{k+1} = \omega \cdot \lambda^k$
   b. $\boldsymbol{u^{k+1}} = \max[0, \boldsymbol{u^k} + \lambda^k \cdot \boldsymbol{f}(\boldsymbol{\mu^k})]$

7. Set $k = k + 1$, and go to step 3.

In step 6.b, $\boldsymbol{f}$ is the original equality constraint (3d) that corresponds to constraint (6b) in the decentralized problem; the operator "max" is to be understood component-wise. The problem solved in Algorithm 3 step 1 does not consider constraint (12b), but it is a bounded problem (i.e., $\boldsymbol{\mu} \in [\boldsymbol{\mu_{lb}}, \boldsymbol{\mu_{ub}}]$). The update of the penalty factor λ_k must define a factor $\omega > 1$. In this study, we used $\omega = 1.1$ as was presented in Kall and Wallace (2008).

As mentioned previously, the following changes must be made to Algorithm 3 to solve the problem of shared resources and the distribution P_D^1:

- Step 3 should not be performed.
- In step 4, $\boldsymbol{\mu^k}$ must be found by solving the bounded decentralized problem:

$$\min_{\boldsymbol{\mu_i^k} \in [\boldsymbol{\mu_{lb}}, \boldsymbol{\mu_{ub}}]} \tilde{L}_i\left(\boldsymbol{\mu}, \boldsymbol{u_i^k}, \lambda^k\right)$$

where i represents an individual sub-unit, and $\boldsymbol{\mu_i^k}$ represents the decision variables for sub-unit i in iteration k in particular. In this case, $\tilde{L}$ is defined as in Equation (11) with the dualized constraint from Equation (9d).

- In step 5, check constraint (5b) as a convergence criterion.

For this particular method of resolution and due to the computation time, it was necessary to initialize Algorithm 3 for the shared resources problem with a feasible point. As mentioned previously, this work proposes a method called *clipping*, which is used in step 1 of Algorithm 3.

Clipping: This method is exclusively for the shared resources problem because it works by coordinating only inequality constraints, and it intends to change the topology of P_C^1 by transforming it into n sub-problems, each of which refers to the control and operation of one unit in particular. The decentralized problem that refers to the i^{th} unit can be formulated when the original linking restriction (1d) is modified by introducing an additional parameter $\overline{R_{T_{Ji}}}$. The modified problem ($P_{d\,i}^1,\ i = 1, .., n$) is shown in Equation (13).

$$\min_{[\boldsymbol{\mu_i},\boldsymbol{\nu_i}]} J_i(\boldsymbol{\mu_i},\boldsymbol{\nu_i}) \qquad i=1,\dots,n \qquad (13a)$$

s.t.

$P_{d\,i}^{1}\left(\overline{R_{T_{Jl}}}\right)$:

$$\boldsymbol{f}_i(\dot{\boldsymbol{x}}_i,\boldsymbol{\nu_i},\boldsymbol{\mu_i},t)=0, \quad i=1,\dots,n \qquad (13b)$$

$$\boldsymbol{g_i}(\dot{\boldsymbol{x}}_i,\boldsymbol{\nu_i},\boldsymbol{\mu_i},t)\le 0, \quad i=1,\dots,n \qquad (13c)$$

$$R_{ji}(\boldsymbol{\mu_{ji}}) \le \overline{R_{T_{Jl}}}, \qquad j=1,\dots,N \qquad (13d)$$

The proposed resolution method finds a feasible point $\boldsymbol{M^f} \coloneqq (\boldsymbol{\mu_1},\dots,\boldsymbol{\mu_n})$ by working iteratively as in Algorithm 4:

ALGORITHM 4

1. Set $k=0$ and $\overline{R_{T_{Jl}}} = R_{T_j}$, $\forall i$.
2. Find $\boldsymbol{\mu_i^k}$ by solving the decentralized problem (P_d^1) for each subunit.

$$\boldsymbol{\mu_i^k} = \arg\min\left[P_{d\,\mathrm{i}}^{1}(\overline{R_{T_{Jl}}} = \overline{\boldsymbol{R_{T_{Jl}}}}^{\boldsymbol{k}})\right], \qquad i=1,\dots,n$$

3. Check constraint (1d). If it is true, the proposed control action by step 1 is feasible and can be implemented (i.e., $\boldsymbol{M^f} \coloneqq \left(\boldsymbol{\mu_1^0},\dots,\boldsymbol{\mu_n^0}\right)$); if it is false, the solution proposed in step 1 cannot be implemented, so go to step 3.
4. Set $k=k+1$.
5. Identify an over-demanded resource by checking $R_{ji}\left(\boldsymbol{\mu_{ji}}\right) > \overline{R_{T_{Jl}}}$ and modify $\overline{R_{T_{Jl}}}^{k}$. For the chosen over-demanded resource j, compute:

$$\overline{\boldsymbol{R}_{\boldsymbol{T_{Jl}}}}^{\boldsymbol{k}} = \boldsymbol{\alpha_{ji}} \cdot \boldsymbol{R_{T_j}}, \qquad \boldsymbol{i=1,\dots,n}$$

where $\alpha_{ji} \in\,]0,1]$ and is defined as:

$$\boldsymbol{\alpha_{ji}} = \frac{\boldsymbol{R_{ji}}(\boldsymbol{\mu_{ji}^{k-1}})}{\boldsymbol{r_j}\left(\boldsymbol{\mu_{ji}^{k-1}}\right)}, \qquad \boldsymbol{i=1,\dots,n}$$

6. Return to step 2.

Note that in step 5, α_{ji} forces the fulfillment of constraint (6d) for the over-demanded resource j in particular; therefore, the algorithm gives a feasible solution in $k \leq n$ iterations, which can be used as a starting point for coordination methods that require feasible solutions, such as that in Shahidi et al. (2015).

Although both methods take advantage of the topology of the general problem, the benefit of selecting the projection-based method is that the problem of feasibility of the solution is solved inherently in the method by the projection operator; each iteration carried out follows a path of feasible points while progressing towards the optimum, unlike the augmented Lagrangian method, where none of the points before the optimum is likely to be feasible because the proper Lagrange multipliers are not available.

6. Results

6.1. Shared resources

In Figure 10, the first line shows the evolution and reference of the temperature, and lines two and three show the control action proposed by each method. The last line shows the consolidated consumption of the resources (the total consumption of a specific resource in the four units simultaneously). The optimization problem with 128 decision variables for the full control horizon (180 (s)) is solved approximately every 11 seconds.

Set points are reached in every case. The resources are distributed intuitively: resource two, which is more expensive, is used only when resource one is not available, or it is saturated and when the system is far from the goal, moreover, the graphs of the consolidated consumption show that the uses of both resources are simultaneously saturated only in this case.

In some sections the distribution of resources is saturated (or has reached its upper bound), especially when a change of the set point is made (i.e., from the point of view of the controller, an energy deficiency is taking place) and when the system is far from the objective. If a resource is saturated, the constraint associated with that resource is active, and the use of any of the proposed methods is validated.

There are negligible differences between the cost functions in Table 1, but the behavior shown in the last column justifies the use of the projection-based method or the augmented Lagrangian method because the calculation time is reduced by up to 90%. It is clear that having a shorter calculation time will cause the control decision to be made more precisely because the temperature that is being used as the current state of the system in the optimization protocol is closer to the real temperature. Therefore, if a projection of the temperature is used until the expected calculation time to try to obtain a more accurate solution, this will be approximated with a minor error; thus, a better decision can be made and applied.

Being an experimental system, although all the graphs are qualitatively equal, slight differences appear in the temperature curves. These differences can be attributed mainly to disturbances (like small changes in the feed quality), because the tests for each resolution method are carried out in series. In a simulated environment, all methods deliver the same result and the curves are the same.

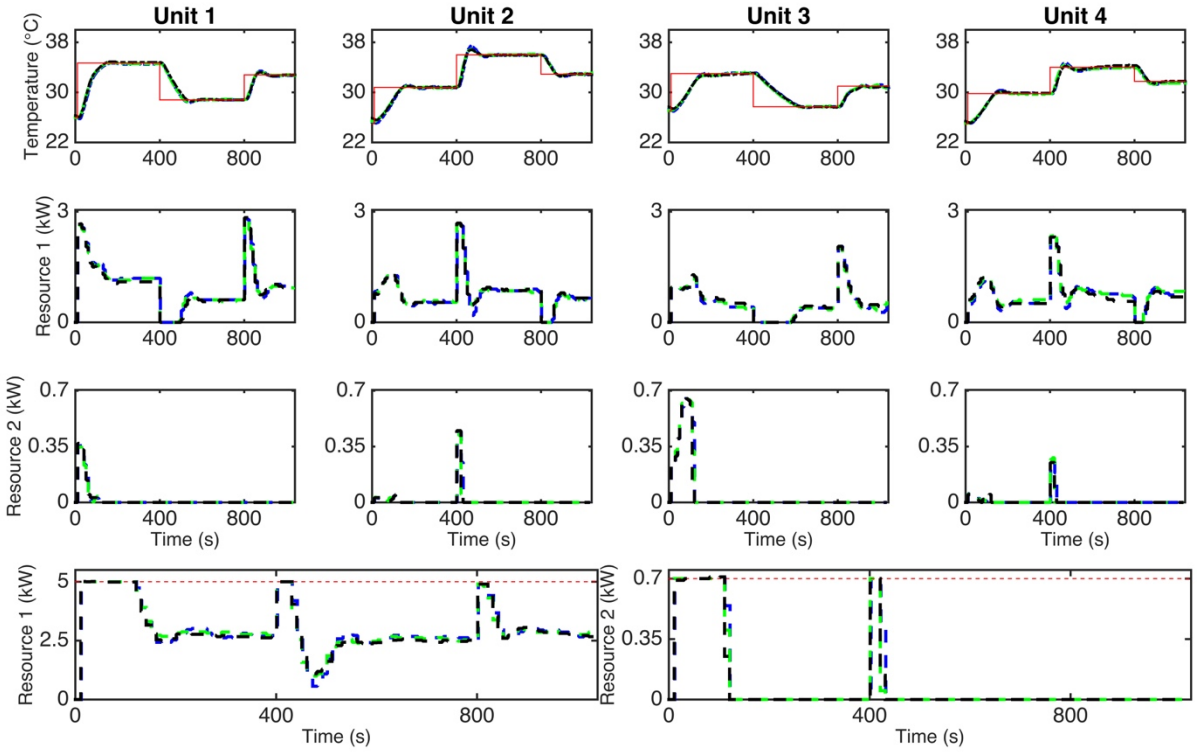

Figure 10: Comparison of the simulation results obtained for the projection-based solution method (black dotted line), the augmented Lagrangian solution method (green dotted line) and the centralized method (blue dotted line). The references are plotted using red dotted lines. The plots in the last row represent the total consumption of every resource (sum of the individual consumptions in every unit).

Table 1: Evaluation of performance indexes for different control schemes in the shared resources problem.

Control scheme	Objective function	Calculation time
Centralized	1.105	7 ± 2 s
Projection	0.985	0.7 ± 0.1 s
Augmented Lagrangian	1.010	4 ± 1 s

6.2. Stochastic Optimization

Figure 11 shows and compares the results obtained for each of the three methods: the black dotted line is the projection method, the green dotted line represents the augmented Lagrangian method, and the blue dotted line is the centralized problem-solving method. Each row represents the performance and control actions in each unit. In Figure 11, the first line shows the evolution and reference of the temperature, whereas lines two and three show the control action proposed by each method. The optimization problem (154 decision variables) for the full control horizon (180 (s)) is solved approximately every 16 seconds.

The set points are reached in every case. As in the previous case, the resources are intuitively distributed. There is a slightly larger difference between the cost functions in Table 2, but the calculation time is reduced by more than 90% in the best scenario. In this problem, the number of decisions parameterized is greater than in the shared resources problem (summarized in Table 1) because the calculation time is generally greater than in the previous problem, and the worst calculation time, approximately 14 (s), was similar to the implementation time τ/θ.

The three methods ensure the resolution of the problems and when evaluating their performance in a receding horizon routine, they practically deliver the same values in the objective function. Considering that in general an optimal control problem must be solved over a limited time span (because otherwise it is inapplicable in the industry), the two approaches proposed are attractive, each with its advantages and limitations.

The improvement in the calculation time of the projection-based method compared to the others can be attributed to the fact that it works progressing from a starting point following a space of feasible points only relying on gradient steps and the operator of projections. The feasibility of the augmented Lagrangian method is subject to the proper multipliers being found, and in each iteration, a sub-optimization routine is solved for each dualized problem.

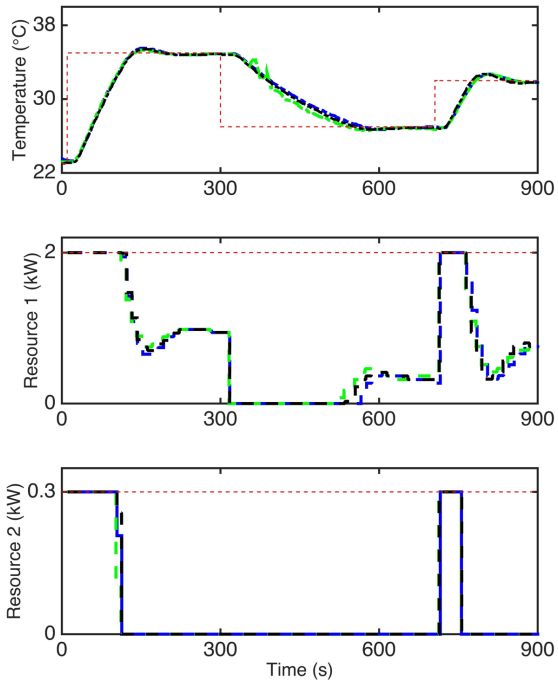

Figure 11: Comparison of the simulation results obtained for the projection-based solution method (black dotted line), the augmented Lagrangian solution method (green dotted line) and the centralized method (blue dotted line). The references are plotted using red dotted lines.

Table 2: Evaluation of performance indexes for different control schemes in the stochastic optimization problem.

Control scheme	**Objective function value**	**Calculation time (s)**
Centralized	1.050	11 ± 3
Projection	0.954	0.9 ± 0.2
Augmented Lagrangian	1.280	5 ± 2

7. Conclusions

An experimental benchmark system was implemented, which effectively allows the evaluation and comparison of large-scale problem-solving strategies with calculation times which are adequate for the problem of optimal control with a receding horizon. This was achieved by measuring temperatures with inexpensive instrumentation.

For the formulation and resolution of large-scale problems (i.e., shared resources and stochastic optimization), as was intuitively expected, the centralized resolution of the original problems delivered the results in the slowest manner.

The other two methods tested had faster calculation times, but they only guarantee convergence to the optimum in an appropriate time under certain conditions.

The method of projections guarantees finding a feasible and optimal solution simultaneously because the problems have an adequate topology; that is, the constraints form a closed, convex and non-empty set. It is the most attractive method in terms of the calculation time.

The augmented Lagrangian method also finds an optimal point, but the calculations must be initialized using a feasible point. This allowed for the proposal of a third method, clipping, because the calculation time is otherwise considerably greater. Although the calculation time is longer than in the projections method, it does not work under the assumption of the convex nature of the restrictions; therefore, it is attractive for problems with other topologies or more general problems because it also reduces the calculation time compared to the original case.

This work presents attractive solutions if it is considered that more complex problems require more efficient algorithms, which has to deliver the correct solutions in a faster way. In addition, a novel method applied to optimal control was presented, such as the use of projections, which solves the problem by iterating.

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Appendix

With the idea of verifying the performance of the proposed methods in an optimal control protocol with a mobile horizon in a real system, an experimental installation has been set up, in which problems similar to the posed can be implemented. The proposed installation has:

- Four units that will act as water heating systems
- Four resistances (helical 2000W/220V) connected to solid state relays (ANV SSR-25LA)
- Four Bunsen Burners
- Four pumps (Masterflex 07528-10, head 77800-52
- Four mechanical agitators (Boeco OSD-20 S65)
- Two water containers
- A gas container
- Four gas mass flow controllers (Cole-Parmer 32907-69) with set point units (Cole-Parmer 32907-85)
- Four electric sparklers (GASFIX IN: 3VDC)
- Four thermocouples (VT-DKSGD-100L-1)
- Two pressure transducers

The computer has a processor Intel(R) Core™ i7-4770 CPU @ 3.40GHz with $8{,}00\ GB$ RAM installed capacity. The instruments, the process lines and the control loops can be seen in Figure 12.

In order to control the resources, monitor the changes in the temperature of each unit and define water supply, the instruments must be connected to a PLC, the equipment used is a Fatek PLC, model FBs-20MAR-2AC. For the proposed system It was necessary to use 5 extra PLC units, which through different communication protocols read information and set the operating conditions proposed by the control system. The communication protocols and the models of each unit are detailed in Table 4. Figures 13 to 16 show photographs of instruments and units already installed in the laboratory.

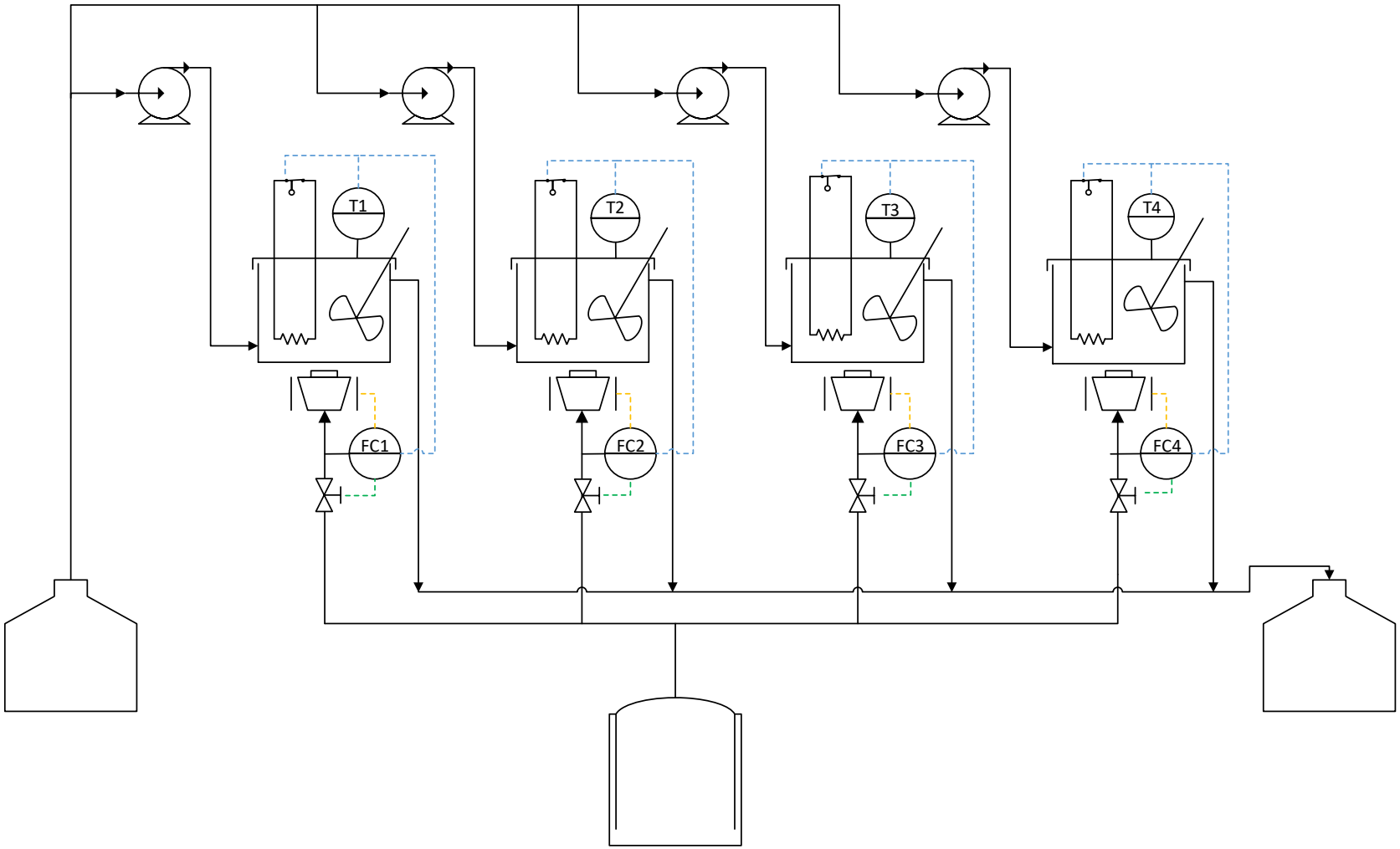

Figure 12: Proposed system diagram

Table 3: models and communication protocol of the units connected to the master PLC.

Unit	Amount of ports	Connected to	Communication protocol
FBs 6TC	6	Thermocouple (4)	0-20 [mA]
FBs 6AD	6	Gas flow meter (4)	0-5 [V]
		Pressure transducer (2)	0-5 [mA]
FBs 4DA	4	Gas flow meter (4)	0-5 [V]
FBs 4DA	4	Pumps (4)	0-10 [V]
FBs 4DA	4	Resistances (4)	0-20 [mA]

Figure 13: implementation of the proposed benchmark system in the laboratory. It can be seen the four units with their respective instrumentation.

Figure 14: Top view of the unit one. It can be seen the agitator, the resistance and the thermocouple.

Figure 15: View of the lighter and its gas mass controller. On the side is the controller's set-point unit.

Figure 16: PLC board (left) and the units required for the communication of all the instruments (order corresponding to Table 3).