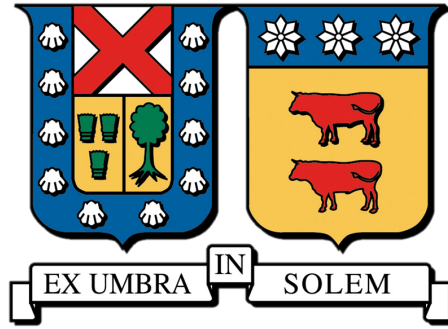




Stripe-Like Domain Nucleation in Synthetic Antiferromagnetic Multilayers Revealed by Soft Spin-Wave Modes

Tesis de pregrado para optar al grado de **Licenciado en Física**.

Departamento de Física,
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*No son más silenciosos los espejos
ni más furtiva el alba aventura;
eres, bajo la luna, esa pantera
que nos es dado divisar de lejos.
Por obra indescifrable de un decreto
divino, te buscamos vanamente;
más remoto que el Ganges y el poniente,
tuya es la soledad, tuyo el secreto.
Tu lomo condesciende a la morosa
caricia de mi mano. Has admitido,
desde esa eternidad que ya es olvido,
el amor de la mano recelosa.
En otro tiempo estás. Eres el dueño
de un ámbito cerrado como un sueño.
—A un gato, Jorge Luis Borges.*

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Abstract

Magnetism is one of the most active and dynamic areas of physics, attracting great interest from scientists in recent decades. This interest relates not only to its relevance in the fundamental understanding of matter but also to its role in the development of new technologies. At the microscopic level, magnetism originates primarily in the intrinsic magnetic moments of atoms, associated with the spin, and the interaction between these moments. These fundamental properties give rise to a variety of collective phenomena, including magnetic ordering in ferromagnetic material and magnetic moment excitations, known as spin waves, which can carry information through the material without generating a net flow of electric charge. This thesis focuses on the investigation of magnetization instability processes in ferromagnetic multilayer systems composed of thin and thick layers. These processes are explored by studying spin wave dynamics in the framework of micromagnetism, particularly through the soft modes, which are specific spin wave modes whose frequencies are close to zero, indicating that the system is approaching a phase transition or magnetic instability. The results indicate that in thin systems, the instability is localized at the surface or within the volume, depending on whether the number of layers is even or odd. Furthermore, when perpendicular anisotropy is considered, instability can occur under certain conditions through the formation of stripe domain textures. As the system becomes thicker, the formation of stripe domains becomes more favored. A phase diagram is obtained by analyzing the soft modes, which enables the prediction of conditions for stripe domain formation. The periodicity of the stripe domains is also determined by the soft mode properties. When the ferromagnetic state is close to destabilization, the stripe domains exhibit chiral properties. In contrast, when the antiferromagnetic state is at the onset of reversion processes, the domains are non-chiral, preferring only one rotation sense. These results offer new insights into predicting the onset of the reversal processes without calculating the complex magnetic textures, but just analyzing the soft spin-wave modes.

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Chapter 1

Introduction

This chapter provides an overview of the various types of magnetic materials that occur naturally. It also introduces the theoretical framework of micromagnetism, detailing the magnetic interactions described by energy densities that will be employed in this work. Finally, the chapter discusses spin-wave band structures and soft modes, establishing the theoretical basis for the calculation and analysis of the results obtained in the subsequent chapters.

1.1 Basic concepts

The basic unit in magnetism is the magnetic moment, which is a vector quantity associated with the torque that an object experiences in the presence of a magnetic field. At a macroscopic level, the magnetic materials can be considered to consist of millions of magnetic moments. Therefore, to describe magnetic properties, the magnetization vector is defined as the density of magnetic moments per unit volume. Different types of magnetism can be observed, including diamagnetism, paramagnetism, ferromagnetism, and antiferromagnetism, among others.

Diamagnetic materials do not contain a permanent magnetic moment, as their atoms consist of paired electrons with opposite spins. In the presence of an external magnetic field, a net magnetization opposite to the field direction is induced. Diamagnetism is present in all substances; however, except in some instances, such as water and noble gases, it typically contributes less significantly than other forms of magnetism [1].

Paramagnetic materials are characterized by a permanent magnetic moment. Unlike diamagnetic materials, their atoms have unpaired electrons, resulting in a local net magnetic moment. In this case, the magnetic moments are uncoupled; thus, in the absence of an external magnetic field, the moments are randomly oriented, resulting in no spontaneous magnetization. By applying an external magnetic field, it then occurs that magnetic moments tend to align themselves parallel to the applied field direction, inducing a net magnetization [1].

Ferromagnetic materials, like paramagnetic materials, possess a permanent magnetic moment, but these moments interact through a short-range interaction known as an exchange interaction [2], in which each moment tries to align its neighboring moments in its direction. This interaction has a quantum origin in the repulsion between electrons and the Pauli exclusion principle. In the absence of external fields, if the ferromagnet is large enough, the magnetic order is usually broken into domains [3, 4] (see Fig. 1.1), regions where all magnetic moments are aligned. Domains are separated by domain walls, where the magnetic moments rotate coherently from the direction in one domain to that in the next. Each domain acts like a tiny magnet that, in equilibrium, orients itself so as to minimize the net magnetic moment of the macroscopic sample. When an external magnetic field is applied, the domains begin to align with the field, resulting in a net magnetization. Some examples of ferromagnetic materials are iron (Fe), nickel (Ni), cobalt (Co), and alloys of these metals, such as permalloy (NiFe).

In **antiferromagnetic** the magnetic moments are linked via exchange interaction, yet they usually align antiparallel and exhibit complete cancellation of magnetic moments. The resulting magnetization is zero. Some examples are transition metal oxides such as MnO, CoO, and NiO.

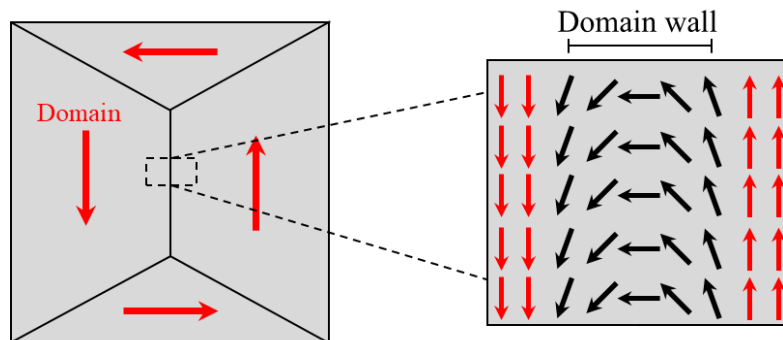


Figure 1.1: The left side shows an illustration of magnetic domains, while the right side presents a representation of the domain wall.

1.2 Micromagnetism

Over time, several models have tried to explain the behavior of ferromagnetic materials. In 1907, Weiss considered the existence of a “molecular field” that attempts to align magnetic moments and introduced the concept of magnetic domains. Still, he was unable to explain the origin of these domains [2]. In 1926, Heisenberg developed a model that incorporates quantum mechanical aspects, which successfully explained the origin of the molecular field as an exchange interaction. In 1935, Landau and Lifshitz took a step further, describing how non-uniform dynamical states of magnetization evolve and providing a continuous description of magnetism.

In 1940, William Fuller Brown developed the theory of micromagnetism, a continuum approach that describes the magnetization processes on significant length scales large enough to replace the discrete nature of magnetic order by a continuous function of position and time. These length scales typically range from nanometers to micrometers for most ferromagnetic materials. This enables the derivation of analytical and semi-analytical solutions that account for magnetic moment distributions and magnetic interactions.

The central approximation lies in the assumption that the spin direction exhibits a smooth variation along the lattice sides, allowing it to be expressed as a continuous function of position and time. So, the macroscopic behavior of the system can be described as a vectorial field, which will correspond to the magnetization $\mathbf{M}(\mathbf{r}, t)$, where $\mathbf{r}$ represents the position vector and t denotes the time.

Under the assumption that the spin operators are replaced by classical vectors, with the additional consideration of the magnetization magnitude remaining constant over time, and the system being at temperatures well below the Curie temperature, it is possible to derive the equation that describes the temporal evolution of the magnetization of a ferromagnetic material. This equation is known as the Landau-Lifshitz (LL) equation of motion, given by [5]

$$\frac{\partial \mathbf{M}(\mathbf{r}, t)}{\partial t} = -\mu_0 \gamma \mathbf{M}(\mathbf{r}, t) \times \mathbf{H}^{\text{eff}}(\mathbf{r}, t), \quad (1.1)$$

where $\mathbf{H}^{\text{eff}}(\mathbf{r}, t)$ is an effective magnetic field, γ is the gyromagnetic constant, and μ_0 is the vacuum permeability.

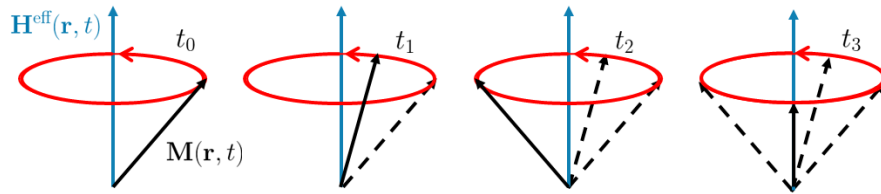


Figure 1.2: Illustration of the temporal evolution of magnetization under the influence of an effective magnetic field.

Equation 1.1 describes the time evolution of magnetization $\mathbf{M}(\mathbf{r}, t)$ under the influence

a magnetic field $\mathbf{H}^{\text{eff}}(\mathbf{r}, t)$,¹ which considers all interactions of the system, namely

$$\mathbf{H}^{\text{eff}} = \mathbf{H}^{\text{Zee}} + \mathbf{H}^{\text{ex}} + \mathbf{H}^{\text{ani}} + \mathbf{H}^{\text{dip}} + \mathbf{H}^{\text{inter ex}}. \quad (1.2)$$

Here, the right terms of Eq. (1.2) correspond to the Zeeman interaction, exchange interaction, uniaxial anisotropy, dipole-dipole interaction, and interlayer exchange interaction, respectively. Each field is derived from the functional derivative of the energy density $\varepsilon(\mathbf{m})$. This is

$$\mathbf{H}^{\text{eff}} = -\frac{1}{\mu_0} \frac{\delta \varepsilon}{\delta \mathbf{m}}. \quad (1.3)$$

Therefore, while the atomistic description of magnetism is well addressed by accounting for the energetic interactions between spins, in the micromagnetism approach, it is more helpful to work with effective fields.

1.3 Magnetic interactions

In what follows, the different interactions related to this thesis are described in the continuous limit.

1.3.1 Zeeman energy

The Zeeman energy describes the interaction between magnetic moments and an external magnetic field $\mathbf{H}_0$ which tries to align them in its direction, as shown in Figure 1.3. In this case, the energy density is given by

$$\varepsilon^{\text{Zee}} = -\mu_0 \mathbf{H}_0 \cdot \mathbf{M}.$$

¹It is worth mentioning that this equation can include a α parameter (called Gilbert damping) which is related to the dissipation of the system. This equation is known as the Landau-Lifshitz-Gilbert (LLG) equation.

$$\frac{\partial \mathbf{M}(\mathbf{r}, t)}{\partial t} = -\mu_0 \gamma \mathbf{M}(\mathbf{r}, t) \times \mathbf{H}^{\text{eff}}(\mathbf{r}, t) + \alpha \frac{\mu_0 \gamma}{M_s} \mathbf{M}(\mathbf{r}, t) \times \frac{\partial \mathbf{M}(\mathbf{r}, t)}{\partial t}.$$

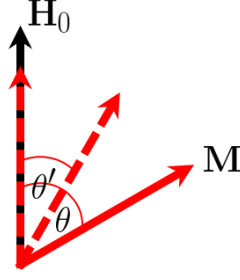


Figure 1.3: Schematic representation of a magnetization vector $\mathbf{M}$ align to the external magnetic field $\mathbf{H}_0$.

1.3.2 Exchange energy

The exchange interaction is the origin of ferromagnetism and is described by the Heisenberg Hamiltonian

$$\mathcal{H}^{\text{ex}} = - \sum_{ij} J_{ij} \mathbf{m}_i \cdot \mathbf{m}_j,$$

where $\mathbf{m}_i$ is the magnetic moment at the site i and J_{ij} is the exchange integral, representing the strength of the coupling between the magnetic moments i and j . Usually, the exchange integral is assumed constant for nearest neighbors, that is, $J_{ij} = J$, and is zero for distant neighbors. If $J > 0$, parallel alignment is favored, while if $J < 0$, antiparallel alignment is favored.

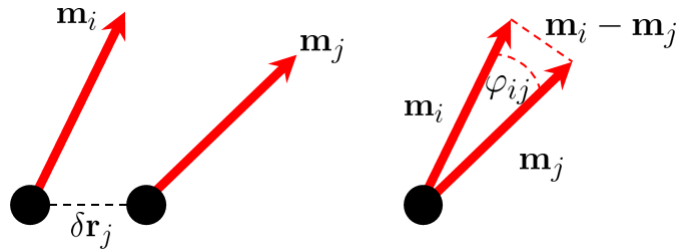


Figure 1.4: Schematic representation of the change in the angle between neighboring magnetic moments i and j separated by a $\delta \mathbf{r}_j$ vector.

In the continuous approximation, the spin operator is considered to be a vector, and the exchange energy is²

$$E^{\text{ex}} = -Jm^2 \sum_{ij} \cos(\varphi_{ij}). \quad (1.4)$$

Here, m is the magnitude of $\mathbf{m}_i$ and φ_{ij} is the angle between neighboring moments as shown in Fig. 1.4. Of course, in the micromagnetic approximation, the angle φ_{ij} is

²In the continuous approach, the Hamiltonian $\mathcal{H}^{\text{ex}}$ is changed by an energy E^{ex} to make a difference between the discrete and continuous cases

assumed to be too small ($|\varphi_{ij}| \ll 1$). Then, expanding the equation (1.4), it is obtained

$$E^{\text{ex}} = -Jm^2 \sum_{ij} \left(1 - \frac{\varphi_{ij}^2}{2} \right) + \mathcal{O}(\varphi_{ij}^3)$$

$$E^{\text{ex}} \approx E_0^{\text{ex}} + \frac{Jm^2}{2} \sum_{ij} \varphi_{ij}^2.$$

The term E_0^{ex} represents the energy of the system when all moments are aligned, so this term is not relevant for the calculations. The remaining term is associated with variations in magnetization. The angle between the magnetization vectors at two sites can be determined by considering the relative orientation of these unit vectors as shown in Figure 1.4. Thus,

$$|\hat{m}_i - \hat{m}_j|^2 = |\hat{m}_i|^2 + |\hat{m}_j|^2 - 2|\hat{m}_i||\hat{m}_j|\cos(\varphi_{ij}),$$

as $|\hat{m}_i| = |\hat{m}_j| = 1$. Then

$$|\hat{m}_i - \hat{m}_j|^2 = 2[1 - \cos(\varphi_{ij})] \approx \varphi_{ij}^2.$$

Now, since $\hat{m}_i$ is a vector parallel to the magnetization at site i and is given by $\hat{m}_i = \hat{m}(\mathbf{r}_i)$ while $\hat{m}_j$ represents the displaced moments and is given by $\hat{m}_j = \hat{m}_i(\mathbf{r}_i + \delta\mathbf{r}_j)$ with $|\delta\mathbf{r}_j| \ll 1$, the first-order Taylor expansion gives

$$|\hat{m}_j - \hat{m}_i| = |\hat{m}_i - \hat{m}_j| = |(\delta\mathbf{r}_j \cdot \nabla)\hat{m}_i|.$$

Then, the exchange energy can be written as

$$E^{\text{ex}} = \frac{Jm^2}{2} \sum_i \sum_j |(\delta\mathbf{r}_j \cdot \nabla)\hat{m}_i|^2.$$

The first sum is over the sites, and the second summation is over the position vectors from the lattice point i to all its neighbors. Changing the summation over i to an integral over all ferromagnetic. For a cubic crystal, the density of exchange energy is given by

$$\varepsilon^{\text{ex}} = \frac{A_{\text{ex}}}{M_s^2} [(\nabla M_x)^2 + (\nabla M_y)^2 + (\nabla M_z)^2],$$

with $A_{\text{ex}} = 2Jm^2(c/a)$ where a is the lattice parameter and $c = 1, 2$ and 4 for a simple cubic, bcc, and fcc, respectively [3].

1.3.3 Anisotropy energy

The Heisenberg Hamiltonian is isotropic; thus, the magnetic moments align with each other, but there is no direction in which they must do so. However, experimental

evidence indicates the presence of a new energy term known as anisotropy energy, which is associated with the preferred directions of magnetization. There are several types of anisotropies; the most common is magnetocrystalline anisotropy, which is related to the material's crystal structure. In this work, only uniaxial anisotropies are considered. The uniaxial anisotropy energy density is given by [2]

$$\varepsilon^u = -\frac{H_u}{2M_s} (\mathbf{M} \cdot \hat{\mathbf{u}})^2,$$

where $\hat{\mathbf{u}}$ is a unit vector that indicates the anisotropy axis, and H_u is the anisotropy field. If $H_u > 0$, the alignment along the direction $\hat{\mathbf{u}}$ is favored, and this direction is referred to as the easy axis. On the other hand, if $H_u < 0$, the orientation perpendicular to $\hat{\mathbf{u}}$ is favored, resulting in an easy plane, and $\hat{\mathbf{u}}$ hence becomes the hard axis.

As an example, figure 1.5 describes how the anisotropy energy exhibits a minimum (easy Axis) and a maximum (hard axis) for different in-plane magnetization directions.

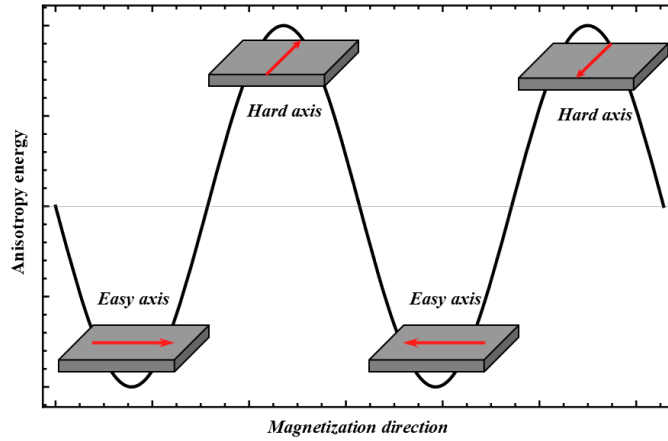


Figure 1.5: Anisotropy energy as a function of the in-plane magnetization direction. The minima in the curve describe the low-energy states, while the maxima denote states of high energy.

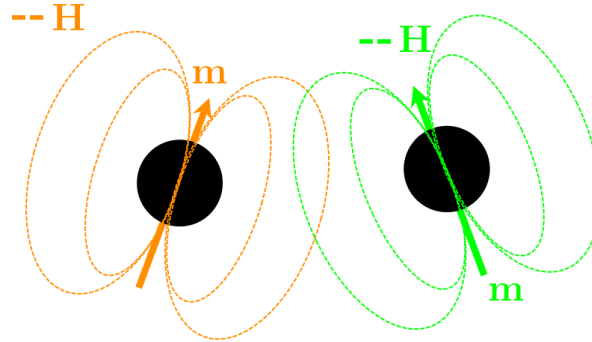


Figure 1.6: Illustration of the interaction between two magnetic moments.

1.3.4 Dipolar energy

The dipolar energy arises from the interaction between dipoles within a magnetic material. A schematic representation of the dipolar fields produced by two dipoles is shown in Fig. 1.6. This interaction is of a long-range nature and depends on the orientations of the dipoles. In a magnetic material, the dipoles or magnetic moments are generated by the spin and the orbital angular momentum. Because there are too many magnetic moments, the dipolar interaction is usually addressed in the continuum approach. In this limit, the dipolar energy density is expressed as

$$\varepsilon^{\text{dip}} = -\frac{\mu_0}{2} \mathbf{M} \cdot \mathbf{H}^{\text{d}},$$

where $\mathbf{H}^{\text{d}}$ is calculated from Maxwell's equations, using a scalar potential ϕ such that $\mathbf{H}^{\text{d}} = -\nabla\phi$. The potential ϕ is given by

$$\phi(\mathbf{r}) = -\frac{1}{4\pi} \iiint_{\mathcal{V}} \frac{\nabla \cdot \mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3r' + \frac{1}{4\pi} \oint_S \frac{\hat{\mathbf{n}} \cdot \mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2r'.$$

Here, $\hat{\mathbf{n}}$ is a normal unit vector on the surface. The first term describes the volumetric magnetic charge distribution, while the second corresponds to the surface magnetic charge, which is induced at the interface between the magnetic and non-magnetic media.

1.3.5 Interlayer exchange interaction

The magnetic coupling between two or more layers (see Fig. 1.7) that are separated by a non-magnetic spacer can be expressed as [6]

$$\varepsilon^{\text{inter}} = -\frac{J_{\text{inter}}}{M_s^{(\nu)} M_s^{(\eta)}} [\mathbf{M}^{(\nu)}(\mathbf{r}) \cdot \mathbf{M}^{(\eta)}(\mathbf{r})], \quad (1.5)$$

with J_{inter} being the interlayer exchange constant between layers and $M_s^{(\nu)}$ is the saturation magnetization of layer ν . This energy term describes how magnetization in one layer interacts with the adjacent layers.

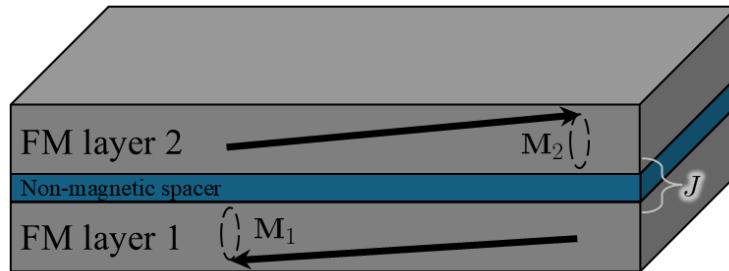


Figure 1.7: A bilayer system composed of two coupled ferromagnetic layers, separated by a non-magnetic spacer.

It is worth mentioning that the interlayer exchange interaction is an interfacial energy. So, in the case of thick films, it must be taken into account that this interaction acts only at the interfaces of the magnetic layers. A good approach to account for this interfacial nature is to split the thick ferromagnetic layer into several sublayers. Then, the interlayer exchange interaction is considered only in the sublayers adjacent to the non-magnetic layer. This approximation is usually referred to as the dynamic matrix method, which is discussed in the next Chapter.

1.4 Spin waves and band structure

Spin waves (SWs) are collective excitations that propagate in a magnetic material. Like mechanical and electromagnetic waves, SWs can be described by a wave vector $\mathbf{k}$ and frequency f . They arise from the collective precession of the magnetic moments around their equilibrium position. An illustration of an SW with wavelength λ is shown in Figure 1.8.

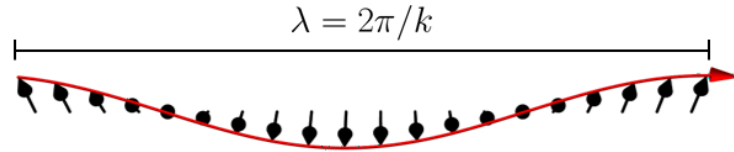


Figure 1.8: 1D spin wave representation.

As electrons in a crystal, spin waves possess a dispersion relation or band structure that describes the relationship between frequency and wave vector, thereby determining their propagation characteristics, such as group and phase velocities. These properties can be extracted from the frequency bands and tell us how fast and how large the wave packet is within the material. Competing interactions, such as anisotropies or Dzyaloshinskii-Moriya interactions (DMI), can modify the shape and curvature of the band structure; for example, they can produce non-reciprocity in wave propagation, flat bands [7, 8], band gaps, or indirect gaps [9] in the case of a periodic structure. The behavior and properties of SW are also highly dependent on the geometry of the nanostructure. Examples of SW dispersions include one- and two- dimensional magnonic crystals [10, 11], superlattices [12, 13], quasicrystals, magnetic nanotubes [14, 15], and bilayers [16, 17], as shown in figure 1.9.

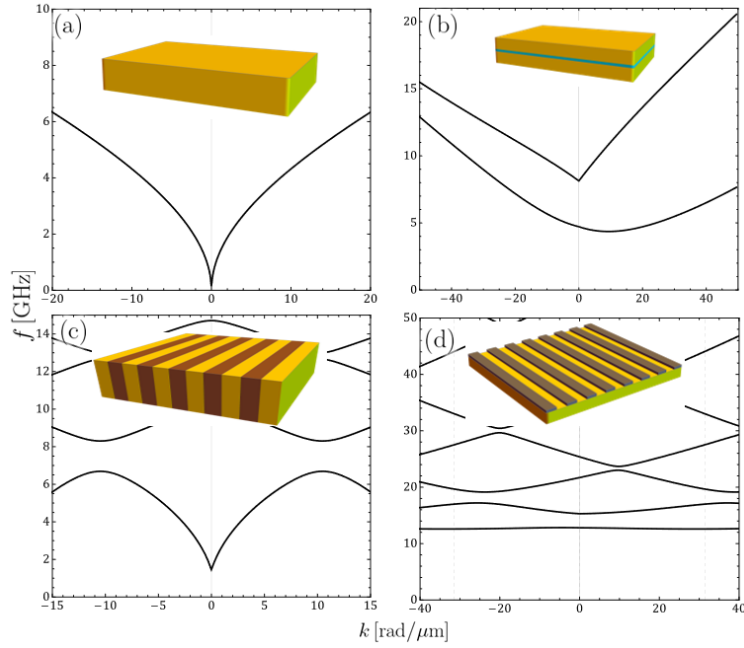


Figure 1.9: Typical SW band structures for (a) a ferromagnetic thin film, (b) a bilayer with magnetizations coupled antiferromagnetically, (c) a conventional magnonic crystal, and (d) a ferromagnetic film coupled with a periodic array of heavy-metal stripes.

1.5 Soft modes

Beyond the well-known advantages of the spin waves, such as broad frequency tunability, nanometric wavelengths, and high group velocities, their investigation provides essential insights into both dynamic and equilibrium magnetic properties. For instance, SWs are highly sensitive to intrinsic magnetic parameters, including anisotropy, exchange coupling, saturation magnetization, dipolar interactions, and surface or interface effects [18]. This sensitivity makes SW spectroscopy a powerful probe for characterizing fundamental material properties and detecting inhomogeneities or impurities. Furthermore, SWs are useful indicators of magnetic phase transitions. Of particular significance are the low-frequency modes, which often exhibit critical behavior near phase boundaries [18, 19]. As external conditions like magnetic field or temperature are varied, these modes can "soften", with their frequencies approaching zero, which is an indication of an impending magnetic instability.

Therefore, the soft modes play a fundamental role in these transition phase processes, as their spatial SW profiles provide information about the emerging spin texture state [20]. Examples of soft modes have been studied in magnetic stripes [21] and in elliptical nanodots [22], ferromagnet/heavy-metal alloys [23], as well as in multilayer systems, where the soft mode exhibits a surface mode nature that implies a magnetization reversal process of the same nature [18, 19]. This surface magnetization reversal process

is illustrated in Fig. 1.10, where the magnetization starts to reverse on top of the multilayer structure. This behavior is anticipated by the soft modes, which possess a similar surface nature. Namely, before the nucleation, such soft modes correspond to magnetization oscillations of the outer top layers.

Hence, soft modes serve as an indicator of phase transitions, particularly for multilayer systems where edges or surfaces play a relevant role. Overall, the theoretical modeling of the static spin textures is complex because, in the case of stripe domains or other textures, the dipolar interaction can be challenging to handle with analytical methods. However, the spin-wave spectrum can be calculated in a simple ground state (such as ferromagnetic), and then, with the properties of the soft modes, crucial information about the impending spin texture can be extracted.

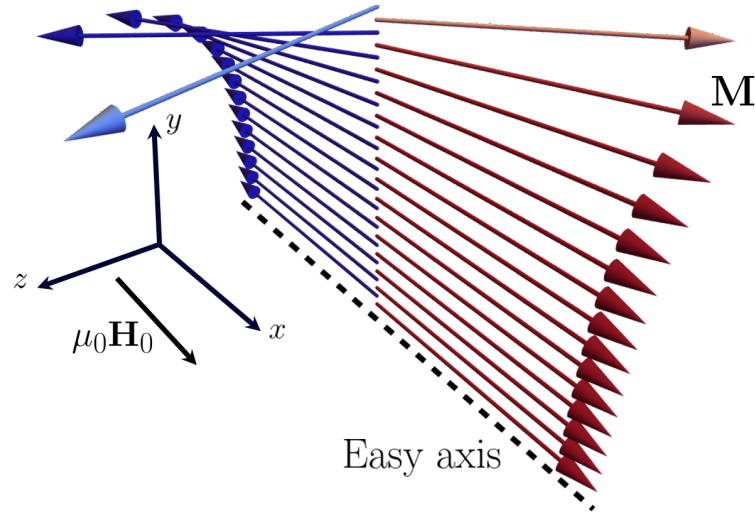


Figure 1.10: Illustration of the onset of the surface reversal processes on an antiferromagnetic multilayer structure. The magnetization pointing along $-x$ starts to reverse at one surface due to the application of an external field along $+x$.

Chapter 2

Theoretical multilayer model

In this chapter, the previous concepts are applied to a multilayer system that considers ferro- and antiferromagnetic coupling between layers. Additionally, each ferromagnetic film is not restricted to being thin, allowing for internal variations of the magnetization dynamics. The equilibrium states are also derived, which will be connected with the soft modes (SW modes calculated at frequencies close to zero) as discussed in Chapter 3. The calculations presented in this chapter were made following the previous works [24, 25].

2.1 Fundamental equations

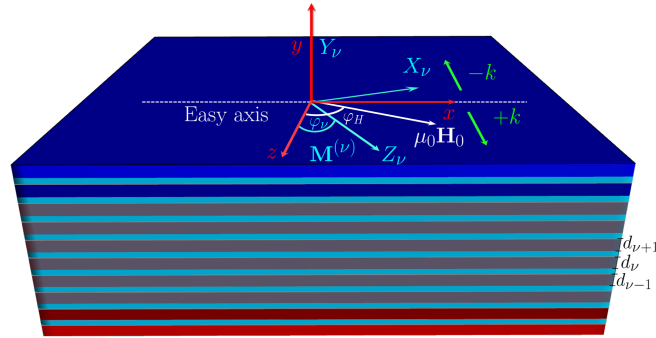


Figure 2.1: Schematic representation of a multilayered structure. The reference system (X_ν, Y_ν, Z_ν) is a local coordinate system, where Z_ν point along the equilibrium magnetization of ν -th layer. The magnetization makes an angle φ_ν with respect to the z axis, which is associated with a general coordinate system (x, y, z) . Each layer has a thickness d_ν . The spin waves propagate along the z -axis. An easy axis pointing in the x direction. Overall, the layers, separated by a spacer of thickness s_ν , may have different thicknesses and magnetic parameters.

To theoretically describe the system, the LL equation is considered for each layer:

$$\frac{\partial \mathbf{M}^{(\nu)}(\mathbf{r}, t)}{\partial t} = -\mu_0 \gamma \mathbf{M}^{(\nu)}(\mathbf{r}, t) \times \mathbf{H}^{\text{eff}(\nu)}(\mathbf{r}, t) \quad (2.1)$$

where the index ν refers to the ν -th layer, $\mathbf{M}^{(\nu)}(\mathbf{r}, t)$ is the magnetization, $\mathbf{H}^{\text{eff}(\nu)}(\mathbf{r}, t)$ is the effective magnetic field which is the sum of the magnetic interactions, γ is the gyromagnetic constant, and μ_0 is the vacuum permeability.

For small deviations around the equilibrium, the magnetization and the effective magnetic field can be written as:

$$\mathbf{M}^{(\nu)}(\mathbf{r}, t) = M_s^{(\nu)}(\mathbf{r}) \hat{\mathbf{Z}} + m_X^{(\nu)}(\mathbf{r}, t) \hat{\mathbf{X}} + m_Y^{(\nu)}(\mathbf{r}, t) \hat{\mathbf{Y}} \quad (2.2)$$

$$\mathbf{H}^{\text{eff}(\nu)}(\mathbf{r}, t) = \mathbf{H}_0^{\text{eff}(\nu)}(\mathbf{r}) + h_X^{\text{eff}(\nu)}(\mathbf{r}, t) \hat{\mathbf{X}} + h_Y^{\text{eff}(\nu)}(\mathbf{r}, t) \hat{\mathbf{Y}} \quad (2.3)$$

with $M_s^{(\nu)}(\mathbf{r})$ is the saturation magnetization and $\mathbf{H}_0^{\text{eff}(\nu)}(\mathbf{r})$ is the static part of the effective field, while $m_{X,Y}^{(\nu)}(\mathbf{r}, t)$ and $h_{X,Y}^{\text{eff}(\nu)}(\mathbf{r}, t)$ represent the dynamics parts of magnetization and magnetic field, respectively.

Assuming that the time dependence is harmonic $m_{X,Y}^{(\nu)}(\mathbf{r}, t) = m_{X,Y}^{(\nu)}(k) e^{i(kz - \omega t)}$. Then, equation 2.1 becomes in

$$-\frac{i\omega}{\mu_0 \gamma} m_X^{(\nu)}(\mathbf{r}) = M_s^{(\nu)} h_Y^{\text{eff}(\nu)}(\mathbf{r}) - H_{0Z}^{\text{eff}(\nu)}(\mathbf{r}) m_Y^{(\nu)}(\mathbf{r}) \quad (2.4)$$

$$-\frac{i\omega}{\mu_0 \gamma} m_Y^{(\nu)}(\mathbf{r}) = -M_s^{(\nu)} h_X^{\text{eff}(\nu)}(\mathbf{r}) + H_{0Z}^{\text{eff}(\nu)}(\mathbf{r}) m_X^{(\nu)}(\mathbf{r}) \quad (2.5)$$

The equilibrium conditions $-M_s^{(\nu)} H_X^{\text{eff}(\nu)} = M_s^{(\nu)} H_Y^{\text{eff}(\nu)} = 0$ must be satisfied. Furthermore, the terms $M_s^{(\nu)} h_Y^{\text{eff}(\nu)}(\mathbf{r})$ and $M_s^{(\nu)} h_X^{\text{eff}(\nu)}(\mathbf{r})$ can be expressed as

$$M_s^{(\nu)} h_X^{\text{eff}(\nu)}(\mathbf{r}) = \sum_{\eta=1}^N \Gamma_{X_\eta}^{X_\nu} m_X^{(\eta)}(\mathbf{r}) + \Gamma_{Y_\eta}^{X_\nu} m_Y^{(\eta)}(\mathbf{r}) \quad (2.6)$$

$$M_s^{(\nu)} h_Y^{\text{eff}(\nu)}(\mathbf{r}) = \sum_{\eta=1}^N \Gamma_{X_\eta}^{Y_\nu} m_X^{(\eta)}(\mathbf{r}) + \Gamma_{Y_\eta}^{Y_\nu} m_Y^{(\eta)}(\mathbf{r}) \quad (2.7)$$

where N is the total number of layers. Finally, equations 2.4 and 2.5 become

$$-\frac{i\omega}{\mu_0 \gamma} m_X^{(\nu)}(k) = -H_{0Z}^{\text{eff}(\nu)}(\mathbf{r}) m_Y^{(\nu)}(k) + \sum_{\eta=1}^N \Gamma_{X_\eta}^{Y_\nu} m_X^{(\eta)}(k) + \Gamma_{Y_\eta}^{Y_\nu} m_Y^{(\eta)}(k) \quad (2.8)$$

$$-\frac{i\omega}{\mu_0 \gamma} m_Y^{(\nu)}(k) = H_{0Z}^{\text{eff}(\nu)}(\mathbf{r}) m_X^{(\nu)}(k) - \sum_{\eta=1}^N \Gamma_{X_\eta}^{X_\nu} m_X^{(\eta)}(k) + \Gamma_{Y_\eta}^{X_\nu} m_Y^{(\eta)}(k) \quad (2.9)$$

Now, equations 2.8 and 2.9 can be expressed as a matrix

$$-\frac{i\omega}{\mu_0 \gamma} \mathbf{m} = \tilde{\mathbf{A}} \mathbf{m}, \quad (2.10)$$

where $\mathbf{m}^T = \{m_X^{(1)}, m_Y^{(1)}, \dots, m_X^{(\nu)}, m_Y^{(\nu)}, \dots, m_X^{(N)}, m_Y^{(N)}\}$ and $\tilde{\mathbf{A}}$ is a block matrix

$$\tilde{\mathbf{A}} = \begin{pmatrix} A_{X^{(\eta)}}^{X^{(\nu)}} & A_{Y^{(\eta)}}^{X^{(\nu)}} \\ A_{X^{(\eta)}}^{Y^{(\nu)}} & A_{Y^{(\eta)}}^{Y^{(\nu)}} \end{pmatrix}. \quad (2.11)$$

The matrix elements need the knowledge of the effective fields. Therefore, in the following section, the derivation of the effective fields for the different magnetic interactions is realized.

2.2 Effective field

For effective field calculations, the magnetization and external magnetic field are assumed to be in the film's plane.

2.2.1 Zeeman field

The external field or Zeeman field is given by (see Figure 2.1)

$$H_Z^{(\nu)} = H_0 \cos(\varphi_H - \varphi_\nu) \quad (2.12)$$

$$H_X^{(\nu)} = H_0 \sin(\varphi_H - \varphi_\nu), \quad (2.13)$$

where φ_H is the angle of the external field and φ_ν is the angle of the magnetization that it forms with the z axis.

2.2.2 Uniaxial anisotropy field

Assuming a uniaxial anisotropy with an easy axis along x for each layer, the energy density is

$$\varepsilon_u^{(\nu)} = -\frac{H_u^{(\nu)}}{2M_s^{(\nu)}} (\mathbf{M}^{(\nu)}(\mathbf{r}) \cdot \hat{x})^2 \quad (2.14)$$

where $\mathbf{M}^{(\nu)}(\mathbf{r}) \cdot \hat{x} = M_Z^{(\nu)} \sin(\varphi_\nu) + m_X^{(\nu)} \cos(\varphi_\nu)$, expanding up to first and second order in $M_Z^{(\nu)}$ and $m_X^{(\nu)}$, respectively, and using $H_i^{(\nu)} = -\frac{1}{\mu_0} \frac{\delta \varepsilon^{(\nu)}}{m_i^{(\nu)}}$, the uniaxial anisotropy field components are

$$\mu_0 H_Z^{u(\nu)} = H_u^{(\nu)} \sin^2(\varphi_\nu) \quad (2.15)$$

$$\mu_0 H_X^{u(\nu)} = H_u^{(\nu)} \cos(\varphi_\nu) \sin(\varphi_\nu) \quad (2.16)$$

$$\mu_0 h_X^{u(\nu)} = \frac{H_u^{(\nu)}}{M_s^{(\nu)}} \cos^2(\varphi_\nu) m_X^{(\nu)} \quad (2.17)$$

2.2.3 Perpendicular anisotropy field

A perpendicular anisotropy interaction is considered in the multilayer system. In this case, the energy density is given by

$$\varepsilon_s^{(\nu)} = -\frac{H_s^{(\nu)}}{2M_s^{(\nu)}} (\mathbf{M}^{(\nu)}(\mathbf{r}) \cdot \hat{y})^2, \quad (2.18)$$

where $\mathbf{M}^{(\nu)} \cdot \hat{y} = m_Y^{(\nu)}$. Expanding up to second order in $m_Y^{(\nu)}$, the only nonzero field component is

$$\mu_0 h_Y^{s(\nu)} = \frac{H_s^{(\nu)}}{M_s^{(\nu)}} m_Y^{(\nu)}. \quad (2.19)$$

2.2.4 Exchange field

The intralayer exchange field is given by

$$\mathbf{H}_{\text{ex}}^{(\nu)}(\mathbf{r}) = \left(\nabla \cdot (\lambda_{\text{ex}}^{(\nu)}(\mathbf{r}))^2 \nabla \right) \mathbf{M}^{(\nu)}(\mathbf{r}),$$

where $\lambda_{\text{ex}}^{(\nu)}(\mathbf{r}) = \sqrt{2A_{\text{ex}}/\mu_0} \left(M_s^{(\nu)} \right)^2$ is the exchange length of the ν th layer, and A_{ex} is the exchange stiffness constant. The exchange field components are

$$\begin{aligned} H_Z^{\text{ex}(\nu)}(z) &= 0 \\ h_X^{\text{ex}(\nu)}(z) &= -(\lambda_{\text{ex}}^{(\nu)})^2 k^2 m_X^{(\nu)}(k) e^{ikz} \\ h_Y^{\text{ex}(\nu)}(z) &= -(\lambda_{\text{ex}}^{(\nu)})^2 k^2 m_Y^{(\nu)}(k) e^{ikz}. \end{aligned}$$

2.2.5 Interlayer exchange field

The interlayer exchange energy density is

$$\varepsilon_{\text{ex}}^{(\nu\eta)} = -J_{\text{inter}} \frac{\mathbf{M}^{(\nu)} \cdot \mathbf{M}^{(\eta)}}{M_s^{(\nu)} M_s^{(\eta)}}.$$

Writing the magnetization components in the local coordinate system of the ν th layer

$$\begin{pmatrix} Z_\eta \\ X_\eta \end{pmatrix} = \begin{pmatrix} \cos(\varphi_\eta - \varphi_\nu) & -\sin(\varphi_\eta - \varphi_\nu) \\ \sin(\varphi_\eta - \varphi_\nu) & \cos(\varphi_\eta - \varphi_\nu) \end{pmatrix} \begin{pmatrix} Z_\nu \\ X_\nu \end{pmatrix}$$

it is obtained

$$\begin{aligned} \varepsilon_{\text{ex}}^{(\nu\eta)} &= -\frac{J_{\text{inter}}}{M_s^{(\nu)} M_s^{(\eta)}} \left(M_s^{(\nu)} M_s^{(\eta)} \cos(\varphi_\eta - \varphi_\nu) + M_s^{(\nu)} m_X^{(\eta)} \sin(\varphi_\eta - \varphi_\nu) \right) - \\ &\quad - \frac{J_{\text{inter}}}{M_s^{(\nu)} M_s^{(\eta)}} \left(-m_X^{(\nu)} M_s^{(\eta)} \sin(\varphi_\eta - \varphi_\nu) + m_X^{(\nu)} m_X^{(\eta)} \cos(\varphi_\eta - \varphi_\nu) + m_Y^{(\nu)} m_Y^{(\eta)} \right). \end{aligned}$$

Therefore, the static components are

$$H_X^{\text{inter}(\nu)} = - \sum_{\eta=1}^N (\delta_{\nu+1}^\eta + \delta_{\nu-1}^\eta) \frac{J_{\text{inter}}}{\mu_0 d_\nu M_s^{(\nu)}} \sin(\varphi_\nu - \varphi_\eta) \quad (2.20)$$

$$H_Z^{\text{inter}(\nu)} = \sum_{\eta=1}^N (\delta_{\nu+1}^\eta + \delta_{\nu-1}^\eta) \frac{J_{\text{inter}}}{\mu_0 d_\nu M_s^{(\nu)}} \cos(\varphi_\eta - \varphi_\nu) \quad (2.21)$$

and the dynamics parts are

$$M_s^{(\nu)} h_X^{\text{inter}(\nu)} = \sum_{\eta=1}^N (\delta_{\nu+1}^\eta + \delta_{\nu-1}^\eta) \frac{J_{\text{inter}}}{\mu_0 d_\nu M_s^{(\eta)}} \cos(\varphi_\eta - \varphi_\nu) m_X^{(\eta)} \quad (2.22)$$

$$M_s^{(\nu)} h_Y^{\text{inter}(\nu)} = \sum_{\eta=1}^N (\delta_{\nu+1}^\eta + \delta_{\nu-1}^\eta) \frac{J_{\text{inter}}}{\mu_0 d_\nu M_s^{(\eta)}} m_Y^{(\eta)}. \quad (2.23)$$

The constant J_{inter} takes into account that within the ν th layer there is an intralayer exchange constant $J_{\text{intra}} = 2A_{\text{ex}}/d_n$ [26], with A_{ex} and d_n being the exchange stiffness constant and the thickness of the n th sublayer, while at the interface across the non-magnetic layer there is an interlayer exchange constant J_{inter} .

2.2.6 Dipolar field

The dipolar field is due to surface and volumetric magnetic charge. To obtain this field, the magnetic potential is considered

$$\phi^{(\nu)}(\mathbf{r}) = \frac{1}{4\pi} \iiint_{V^{(\nu)}} \frac{\rho^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' + \frac{1}{4\pi} \iint_{S^{(\nu)}} \frac{\sigma^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}' \quad (2.24)$$

where $\rho^{(\nu)}(\mathbf{r}) = -\nabla \cdot \mathbf{M}^{(\nu)}(\mathbf{r})$ is the volumetric density charge and $\sigma^{(\nu)}(\mathbf{r}) = \hat{\mathbf{n}} \cdot \mathbf{M}^{(\nu)}(\mathbf{r})$ is the surface density charge. Then, the volumetric part is

$$\nabla \cdot \mathbf{M}^{(\nu)}(\mathbf{r}) = \frac{\partial M_z^{(\nu)}(\mathbf{r}, t)}{\partial z}$$

where $M_z^{(\nu)}(\mathbf{r}, t) = M_Z^{(\nu)} \cos(\varphi_\nu) - m_X^{(\nu)}(\mathbf{r}, t) \sin(\varphi_\nu)$ and $m_{X,Y}^{(\nu)}(\mathbf{r}, t) = m_{X,Y}^{(\nu)}(k) e^{i(kz - \omega t)}$, then the volumetric density charge is¹

$$\rho^{(\nu)}(\mathbf{r}) = i \sin(\varphi_\nu) m_X^{(\nu)}(k) k e^{ikz}. \quad (2.25)$$

Replacing equation (2.25) in the first term of (2.24) gives

$$\iiint_{V^{(\nu)}} \frac{\rho^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' = i \sin(\varphi_\nu) m_X^{(\nu)}(k) k \iiint_{V^{(\nu)}} \frac{e^{ikz'}}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}'. \quad (2.26)$$

¹The temporary part will be ignored as it does not affect the calculations

Using $x - x' = x''$ and $z - z' = z''$, the integral becomes²

$$\iiint_{\mathcal{V}^{(\nu)}} \frac{\rho^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' = i \sin(\varphi_\nu) m_X^{(\nu)}(k) k e^{ikz} \int_{-\infty}^{\infty} \int_{y_\nu}^{\infty} \int_{-\infty}^{\infty} \frac{e^{-ikz''}}{\sqrt{(x'')^2 + (y - y')^2 + (z'')^2}} dx'' dy' dz'' \quad (2.27)$$

and if a and $b \in \mathbb{R}$ then [17]

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{iaz}}{\sqrt{x^2 + b^2 + z^2}} dx dz = 2\pi \frac{e^{-|b||a|}}{|a|} \quad (2.28)$$

Therefore, equation (2.27) is given by

$$\iiint_{\mathcal{V}^{(\nu)}} \frac{\rho^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' = 2\pi i \sin(\varphi_\nu) m_X^{(\nu)}(k) k e^{ikz} \int_{y_\nu}^{\infty} \frac{e^{-|k||y-y'|}}{|k|} dy'. \quad (2.29)$$

According with the multilayer system, inside of ν -th layer it is obtained that³

$$\begin{aligned} \int_{y_\nu}^{\infty} \frac{e^{-|k||y-y'|}}{|k|} dy' &= \int_{\sum_{j=1}^{\nu-1} d_j + s_j}^y \frac{e^{-|k|(y-y')}}{|k|} dy' + \int_y^{d_\nu + \sum_{j=1}^{\nu-1} d_j + s_j} \frac{e^{|k|(y-y')}}{|k|} dy' \\ &= 2 \frac{1 - \cosh(|k|\xi_\nu(y)) e^{-|k|\frac{d_\nu}{2}}}{|k|^2}, \end{aligned}$$

where $\xi_\nu(y) = y + \frac{d_\nu}{2} - \sum_{j=1}^{\nu} d_j + s_{j-1}$. Now, outside of the layer, the integral of equation (2.29) requires the consideration $y - y' > 0$. Thus,

$$\begin{aligned} \int_{y_\nu}^{\infty} \frac{e^{-|k||y-y'|}}{|k|} dy' &= \int_{\sum_{j=1}^{\nu-1} d_j + s_j}^{d_\nu + \sum_{j=1}^{\nu-1} d_j + s_j} \frac{e^{-|k|(y-y')}}{|k|} dy' \\ &= 2 \frac{\sinh(|k|\frac{d_\nu}{2}) e^{-|k|\xi_\nu(y)}}{|k|^2}. \end{aligned}$$

For case $y - y' < 0$,

$$\begin{aligned} \int_{y_\nu}^{\infty} \frac{e^{-|k||y-y'|}}{|k|} dy' &= \int_{\sum_{j=1}^{\nu-1} d_j + s_j}^{d_\nu + \sum_{j=1}^{\nu-1} d_j + s_j} \frac{e^{|k|(y-y')}}{|k|} dy' \\ &= 2 \frac{\sinh(|k|\frac{d_\nu}{2}) e^{|k|\xi_\nu(y)}}{|k|^2}. \end{aligned}$$

Finally, the volumetric part of the magnetic potential is

$$\frac{1}{4\pi} \iiint_{\mathcal{V}^{(\nu)}} \frac{\rho^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' = \frac{i \sin(\varphi_\nu) m_X^{(\nu)}(k) k e^{ikz}}{|k|^2} P^{(\nu)}(y, k) \quad (2.30)$$

where the function $P^{(\nu)}(y, k)$ is given by

$$P^{(\nu)}(y, k) = \begin{cases} 1 - \cosh(|k|\xi_\nu(y)) e^{-|k|\frac{d_\nu}{2}} & \text{Inside the } \nu\text{-th layer} \\ \sinh(|k|\frac{d_\nu}{2}) e^{-|k|\xi_\nu(y)} & \text{Outside the } \nu\text{-th layer} \end{cases} \quad (2.31)$$

²The integral $\int_{y_\nu}$ refers to the fact that the integration is over the thickness of the ν -th layer.

³If $\nu = 1$ then $\sum_{j=1}^{\nu-1} d_j + s_j = 0$

Now, the surface part of the magnetic potential is given by

$$\begin{aligned} \iint_{S^{(\nu)}} \frac{\hat{\mathbf{n}} \cdot \mathbf{M}^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}' &= \iint_{S^{(\nu)}} \frac{m_Y^{(\nu)}(k) e^{ikz'}}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}'_{y'=d_\nu+\sum_{j=1}^{\nu-1} d_j+s_j} \\ &\quad - \iint_{S^{(\nu)}} \frac{m_Y^{(\nu)}(k) e^{ikz'}}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}'_{y'=\sum_{j=1}^{\nu-1} d_j+s_j}. \end{aligned}$$

Using the result given in Eq. (2.28), the integrals give

$$\begin{aligned} \iint_{S^{(\nu)}} \frac{e^{ikz'}}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}'_{y'=d_\nu+\sum_{j=1}^{\nu-1} d_j+s_j} &= 2\pi \frac{e^{|k|(y-d_\nu-\sum_{j=1}^{\nu-1} d_j+s_j)}}{|k|} \\ \iint_{S^{(\nu)}} \frac{e^{ikz'}}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}'_{y'=\sum_{j=1}^{\nu-1} d_j+s_j} &= 2\pi \frac{e^{-|k|(y-\sum_{j=1}^{\nu-1} d_j+s_j)}}{|k|}. \end{aligned}$$

Therefore, the surface part is

$$\frac{1}{4\pi} \iint_{S^{(\nu)}} \frac{\hat{\mathbf{n}} \cdot \mathbf{M}^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}' = \frac{m_Y^{(\nu)}(k) e^{ikz}}{|k|} \sinh(|k|\xi_\nu(y)) e^{-|k|\frac{d_\nu}{2}} \quad (2.32)$$

Now, outside of the ν -th layer, the surface integral has to be evaluated for the case $y - y' > 0$. This is

$$\begin{aligned} \iint_{S^{(\nu)}} \frac{e^{ikz'}}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}'_{y'=d_\nu+\sum_{j=1}^{\nu-1} d_j+s_j} &= 2\pi \frac{e^{-|k|(y-d_\nu-\sum_{j=1}^{\nu-1} d_j+s_j)}}{|k|} \\ \iint_{S^{(\nu)}} \frac{e^{ikz'}}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}'_{y'=\sum_{j=1}^{\nu-1} d_j+s_j} &= 2\pi \frac{e^{-|k|(y-\sum_{j=1}^{\nu-1} d_j+s_j)}}{|k|} \\ \frac{1}{4\pi} \iint_{S^{(\nu)}} \frac{\hat{\mathbf{n}} \cdot \mathbf{M}^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}' &= \frac{m_Y^{(\nu)}(k) e^{ikz}}{|k|} \sinh\left(|k|\frac{d_\nu}{2}\right) e^{-|k|\xi_\nu(y)}. \end{aligned}$$

Similarly, for the case $y - y' < 0$ it is obtained

$$\begin{aligned} \iint_{S^{(\nu)}} \frac{e^{ikz'}}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}'_{y'=d_\nu+\sum_{j=1}^{\nu-1} d_j+s_j} &= 2\pi \frac{e^{|k|(y-d_\nu-\sum_{j=1}^{\nu-1} d_j+s_j)}}{|k|} \\ \iint_{S^{(\nu)}} \frac{e^{ikz'}}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}'_{y'=\sum_{j=1}^{\nu-1} d_j+s_j} &= 2\pi \frac{e^{|k|(y-\sum_{j=1}^{\nu-1} d_j+s_j)}}{|k|} \\ \frac{1}{4\pi} \iint_{S^{(\nu)}} \frac{\hat{\mathbf{n}} \cdot \mathbf{M}^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}' &= -\frac{m_Y^{(\nu)}(k) e^{ikz}}{|k|} \sinh\left(|k|\frac{d_\nu}{2}\right) e^{|k|\xi_\nu(y)}. \end{aligned}$$

Finally, the surface part of the magnetic potential is

$$\iint_{S^{(\nu)}} \frac{\hat{\mathbf{n}} \cdot \mathbf{M}^{(\nu)}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2\mathbf{r}' = \frac{m_Y^{(\nu)}(k) e^{ikz}}{|k|} F^{(\nu)}(y, k), \quad (2.33)$$

where the function $F^{(\nu)}(y, k)$ is given by

$$F^{(\nu)}(y, k) = \begin{cases} \sinh(|k|\xi_\nu(y)) e^{-|k|\frac{d_\nu}{2}} & \text{Inside the } \nu\text{-th layer} \\ \text{sign}(\xi_\nu(y)) \sinh(|k|\frac{d_\nu}{2}) e^{|k|\xi_\nu(y)} & \text{Outside the } \nu\text{-th layer} \end{cases} \quad (2.34)$$

Therefore, the magnetic potential is

$$\phi^{(\nu)}(y, z) = \frac{i \sin(\varphi_\nu) m_X^{(\nu)}(k) k e^{ikz}}{|k|^2} P^{(\nu)}(y, k) + \frac{m_Y^{(\nu)}(k) e^{ikz}}{|k|} F^{(\nu)}(y, k). \quad (2.35)$$

The dynamic dipolar field components can be obtained from $H_i^{(\nu)} = -\partial_i \phi^{(\nu)}(y, z)$, so that

$$\begin{aligned} h_z^{(\nu)}(y, z) &= - \left(-\sin(\varphi_\nu) m_X^{(\nu)}(k) P^{(\nu)}(y, k) + i \operatorname{sign}(k) m_Y^{(\nu)}(k) F^{(\nu)}(y, k) \right) e^{ikz} \\ h_y^{(\nu)}(y, z) &= - \left(\frac{\operatorname{sign}(k)}{|k|} i \sin(\varphi_\nu) m_X^{(\nu)} \frac{\partial P^{(\nu)}(y, k)}{\partial y} + \frac{m_Y^{(\nu)}(k)}{|k|} \frac{\partial F^{(\nu)}(y, k)}{\partial y} \right) e^{ikz}. \end{aligned}$$

To take into account the interaction between layers, the stray fields produced by layer ν are averaged into the thickness of layer η . Namely

$$\langle f^{(\nu)} \rangle_{(\eta)} = \frac{1}{d_\eta} \int_{\sum_{j=1}^\eta d_j + s_{j-1} - d_\eta}^{\sum_{j=1}^\eta d_j + s_{j-1}} f^{(\nu)}(y, k) dy. \quad (2.36)$$

Then, the dipolar components are

$$\begin{aligned} h_X^{(\nu)}(z) &= \sum_{\eta=1}^N \left(-\sin(\varphi_\eta) \sin(\varphi_\nu) m_X^{(\eta)}(k) \langle P^{(\nu)} \rangle_{(\eta)} + i \operatorname{sign}(k) \sin(\varphi_\nu) m_Y^{(\eta)}(k) \langle F^{(\nu)} \rangle_{(\eta)} \right) e^{ikz} \\ h_Y^{(\nu)}(z) &= - \sum_{\eta=1}^N \left(\frac{\operatorname{sign}(k)}{|k|} i \sin(\varphi_\eta) m_X^{(\eta)} \left\langle \frac{\partial P^{(\nu)}}{\partial y} \right\rangle_{(\eta)} + \frac{m_Y^{(\eta)}(k)}{|k|} \left\langle \frac{\partial F^{(\nu)}}{\partial y} \right\rangle_{(\eta)} \right) e^{ikz}, \end{aligned}$$

where the following functions are defined by

$$\langle P^{(\nu)} \rangle_{(\eta)} = \begin{cases} 1 - \frac{2}{|k| d_\nu} \sinh\left(|k| \frac{d_\nu}{2}\right) e^{-|k| \frac{d_\nu}{2}} & \text{if } \nu = \eta \\ \frac{2}{|k| d_\eta} \sinh\left(|k| \frac{d_\nu}{2}\right) \sinh\left(|k| \frac{d_\eta}{2}\right) e^{-|k| |\tau_\eta - \tau_\nu|} & \text{if } \nu \neq \eta \end{cases} \quad (2.37)$$

$$\left\langle \frac{\partial P^{(\nu)}}{\partial y} \right\rangle_{(\eta)} = \begin{cases} 0 & \text{if } \nu = \eta \\ \operatorname{sign}(\nu - \eta) \frac{2}{d_\eta} \sinh\left(|k| \frac{d_\nu}{2}\right) \sinh\left(|k| \frac{d_\eta}{2}\right) e^{-|k| |\tau_\eta - \tau_\nu|} & \text{if } \nu \neq \eta \end{cases} \quad (2.38)$$

$$\langle F^{(\nu)} \rangle_{(\eta)} = \begin{cases} 0 & \text{if } \nu = \eta \\ \operatorname{sign}(\eta - \nu) \frac{2}{|k| d_\eta} \sinh\left(|k| \frac{d_\nu}{2}\right) \sinh\left(|k| \frac{d_\eta}{2}\right) e^{-|k| |\tau_\eta - \tau_\nu|} & \text{if } \nu \neq \eta \end{cases} \quad (2.39)$$

$$\left\langle \frac{\partial F^{(\nu)}}{\partial y} \right\rangle_{(\eta)} = \begin{cases} \frac{2}{d_\nu} \sinh\left(|k| \frac{d_\nu}{2}\right) e^{-|k| \frac{d_\nu}{2}} & \text{if } \nu = \eta \\ -\frac{2}{d_\eta} \sinh\left(|k| \frac{d_\nu}{2}\right) \sinh\left(|k| \frac{d_\eta}{2}\right) e^{-|k| |\tau_\eta - \tau_\nu|} & \text{if } \nu \neq \eta \end{cases} \quad (2.40)$$

Here $\tau_\nu = -\frac{d_\nu}{2} + \sum_{i=1}^\nu d_i + s_{i-1}$

2.3 Equilibrium equations

In order to find the equilibrium position, it is necessary to solve the system of coupled equations of the in-plane static field, which meets the condition $H_X^{\text{eff}(\nu)} = H_X^{(\nu)} + H_X^{\text{u}(\nu)} + H_X^{\text{inter}(\nu)} = 0$. Thus,

$$\begin{aligned}
H_0 \sin(\varphi_H - \varphi_1) + \frac{H_u^{(1)}}{\mu_0} \cos(\varphi_1) \sin(\varphi_1) - \frac{J_{\text{inter}}}{\mu_0 d_1 M_s^{(1)}} \sin(\varphi_1 - \varphi_2) &= 0 \\
H_0 \sin(\varphi_H - \varphi_2) + \frac{H_u^{(2)}}{\mu_0} \cos(\varphi_2) \sin(\varphi_2) - \frac{J_{\text{inter}}}{\mu_0 d_2 M_s^{(2)}} \sin(\varphi_2 - \varphi_3) - \frac{J_{\text{inter}}}{\mu_0 d_2 M_s^{(2)}} \sin(\varphi_2 - \varphi_1) &= 0 \\
\vdots & \\
H_0 \sin(\varphi_H - \varphi_N) + \frac{H_u^{(N)}}{\mu_0} \cos(\varphi_N) \sin(\varphi_N) - \frac{J_{\text{inter}}}{\mu_0 d_N M_s^{(N)}} \sin(\varphi_N - \varphi_{N-1}) &= 0.
\end{aligned} \tag{2.41}$$

The number N represents the total number of layers in the system.

2.4 Dynamic matrix

Now, with the expressions of the effective fields, the matrix elements of Eq. (2.11) are derived. For the case $\nu = \eta$, the matrix elements are given by

$$A_{X^{(\nu)}}^{X^{(\nu)}} = 0 \tag{2.42}$$

$$A_{Y^{(\nu)}}^{X^{(\nu)}} = -H_{0Z}^{\text{eff}(\nu)} - k^2 (\lambda_{\text{ex}}^{(\nu)})^2 M_s^{(\nu)} - \frac{M_s^{(\nu)}}{|k|} \left\langle \frac{\partial F^{(\nu)}}{\partial y} \right\rangle_{(\nu)} + H_s^{(\nu)} \tag{2.43}$$

$$A_{X^{(\nu)}}^{Y^{(\nu)}} = H_{0Z}^{\text{eff}(\nu)} + k^2 (\lambda_{\text{ex}}^{(\nu)})^2 M_s^{(\nu)} - H_u^{(\nu)} \cos^2(\varphi_\nu) + \sin^2(\varphi_\nu) M_s^{(\nu)} \langle P^{(\nu)} \rangle_{(\nu)} \tag{2.44}$$

$$A_{Y^{(\nu)}}^{Y^{(\nu)}} = 0. \tag{2.45}$$

where $H_{0Z}^{\text{eff}(\nu)}$ is given by

$$H_{0Z}^{\text{eff}(\nu)} = H_0 \cos(\varphi_H - \varphi_\nu) + H_u^{(\nu)} \sin^2(\varphi_\nu) + \sum_{\eta=1}^N (\delta_{\nu+1}^\eta + \delta_{\nu-1}^\eta) \frac{J_{\text{inter}}}{\mu_0 d_\nu M_s^{(\nu)}} \cos(\varphi_\eta - \varphi_\nu) \tag{2.46}$$

These terms are of an intralayer nature. That means that they are associated with the interactions acting only on layer ν . The interlayer interactions are described through

the matrix elements with the condition $\nu \neq \eta$. In this case, the matrix terms are

$$A_{X(\eta)}^{X(\nu)} = -i \frac{\text{sign}(k)}{|k|} \sin(\varphi_\eta) M_s^{(\nu)} \left\langle \frac{\partial P^{(\nu)}}{\partial y} \right\rangle_{(\eta)} \quad (2.47)$$

$$A_{Y(\eta)}^{X(\nu)} = -\frac{M_s^{(\nu)}}{|k|} \left\langle \frac{\partial F^{(\nu)}}{\partial y} \right\rangle_{(\eta)} + (\delta_{\nu+1}^\eta + \delta_{\nu-1}^\eta) \frac{J_{\text{inter}}}{d_\eta M_s^{(\nu)}} \quad (2.48)$$

$$A_{X(\eta)}^{Y(\nu)} = -(\delta_{\nu+1}^\eta + \delta_{\nu-1}^\eta) \frac{J_{\text{inter}}}{d_\eta M_s^{(\nu)}} \cos(\varphi_\nu - \varphi_\eta) + \sin(\varphi_\nu) \sin(\varphi_\eta) M_s^{(\nu)} \langle P^{(\nu)} \rangle_{(\eta)} \quad (2.49)$$

$$A_{Y(\eta)}^{Y(\nu)} = -i \text{sign}(k) \sin(\varphi_\nu) M_s^{(\nu)} \langle F^{(\nu)} \rangle_{(\eta)}. \quad (2.50)$$

As expected, these terms are of a dipolar and interlayer exchange nature. Because only these energetic terms allow the interaction between different layers.

Chapter 3

Results and discussion

3.1 Simple multilayer synthetic antiferromagnet

In the following results, standard magnetic parameters of permalloy are considered, i.e, $M_s = 750$ kA/m, $A_{\text{ex}} = 10$ pJ/m, and $\gamma = 0.01856$ GHz/G. The antiparallel alignment is induced by a negative interlayer coupling $J_{\text{inter}} = -0.1$ mJ/m² and the thicknesses of the spacers are $s = 1$ nm.

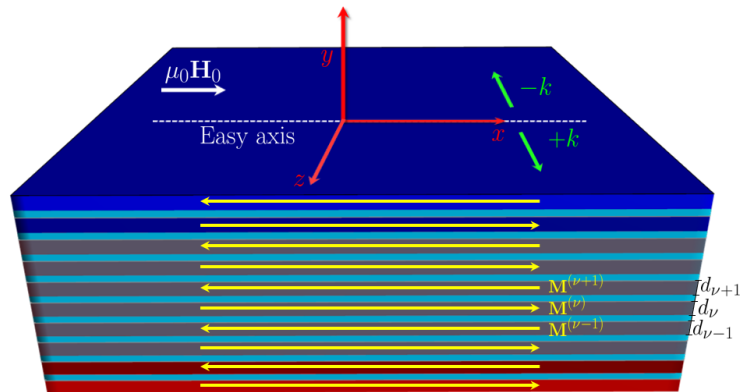


Figure 3.1: Schematic representation of a multilayered synthetic antiferromagnet (SAF), where the equilibrium magnetization points alternately between $-x$ and $+x$. An external magnetic field is applied along the easy axis.

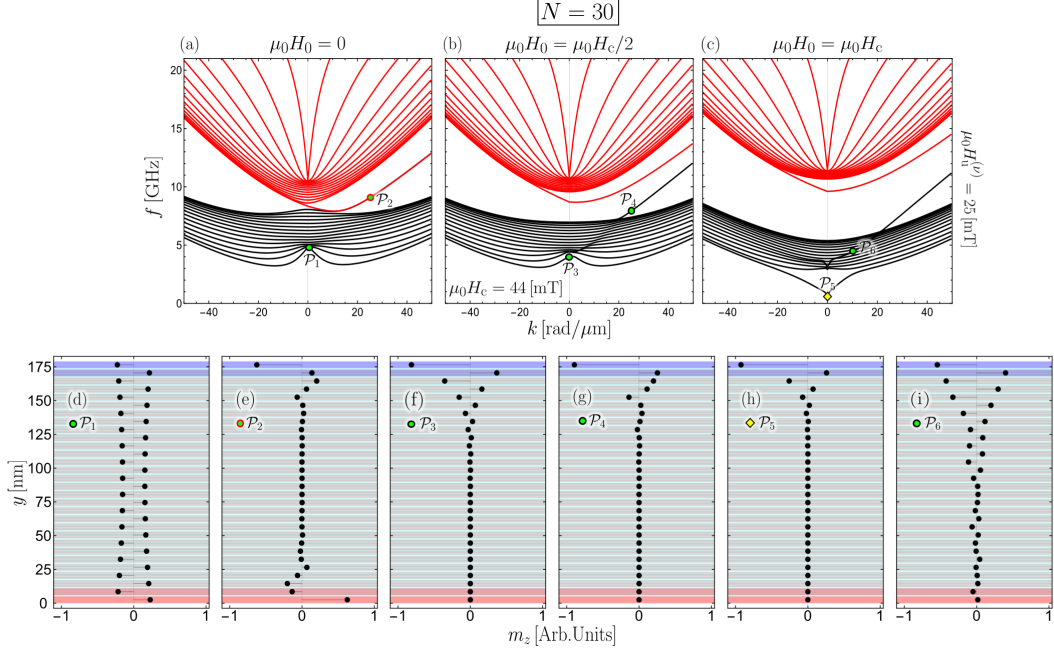


Figure 3.2: (a)-(c) shows the SW band structure of the simple SAF multilayer composed of $N = 30$ layers, where an anisotropy field of $\mu_0 H_u^{(\nu)} = 25$ mT is used. (d)-(i) the SW dynamic profiles of the in-plane m_z component along the thickness.

The first part of the discussion focuses on the simplified case of a synthetic antiferromagnetic multilayer composed of identical thin ferromagnetic layers as shown in Fig. 3.1. In this scenario, each layer has a thickness of $d_\nu = 5$ nm, a uniaxial anisotropy field of $\mu_0 H_u^{(\nu)} = 25$ mT and without a perpendicular anisotropy field $\mu_0 H_s^{(\nu)} = 0$. Figures 3.2(a)-(c) illustrate how the SW band structure evolves in relation to an external magnetic field for $N = 30$. As seen in figure 3.2(a), at zero field and with the magnetizations oriented antiparallel, a distinct surface mode appears as a non-reciprocal isolated band located between the bulk SW bands at positive wave vectors (some bands are highlighted by red and black colors for better visibility). As the external magnetic field increases, this surface mode exhibits a greater frequency reduction than the other modes. Upon reaching a critical field ($\mu_0 H_c = 44$ mT), the frequency approaches zero, indicating that the antiparallel alignment of the magnetizations is not stable anymore. The nature of the modes is shown in the magnetization profiles illustrated in Figs. 3.2(d)-(i), where the dynamic nature of selected modes $\mathcal{P}_{1-6}$ is depicted through the in-plane dynamic magnetization. The profile in Fig. 3.2(d) shows a bulk-like behavior, while Figure 3.2(e) corresponds to a surface mode. As the field increases, the surface mode “penetrates” into the low-frequency bulk bands as shown in Fig. 3.2(b). The point $\mathcal{P}_3$ (Fig. 3.2(f)) exhibits a surface-like behavior similar to that of point $\mathcal{P}_4$ (Fig. 3.2(g)). The spatial profile in Fig. 3.2(h) confirms that the soft mode is characterized by a large amplitude of magnetization oscillations located on the top surface of the multilayer.

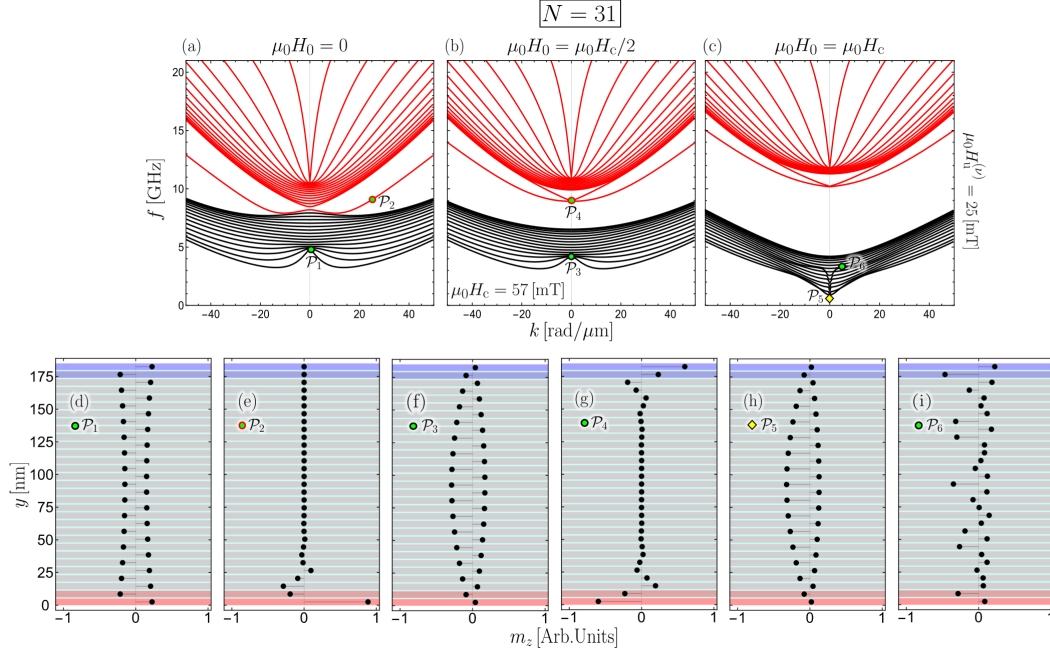


Figure 3.3: (a)-(c) shows the SW band structure of the simple SAF multilayer composed of $N = 31$ layers, where an anisotropy field of $\mu_0 H_u^{(\nu)} = 25$ mT is used. (d)-(i) illustrated the SW dynamic profiles in-plane m_z along the thickness.

Now, the odd case is analyzed. Figs. 3.3(a)-(c) show the spin-wave band structure for a multilayer structure composed of $N = 31$ layers and with $\mu_0 H_u^{(\nu)} = 25$ mT. A localized reciprocal band appears between the bulk bands. As the field increases, it can be seen that this characteristic mode remains at high frequencies, while the low-frequency bulk bands approach zero when the critical field is reached ($\mu_0 H_c = 57$ mT). As revealed by the SW profile illustrated in Fig. 3.3(h), the soft mode now corresponds to a bulk spin-wave profile.

The cases of $N = 30$ (even) and $N = 31$ (odd) demonstrate that the mechanism governing the reversal process (whether it originates at the surface or within the bulk) is influenced by the parity of the layer number, as the system transitions from the antiparallel state to the spin-flop regime [27]. To investigate the connection between the soft modes and the formation of a new spin texture when the external field exceeds the critical field $\mu_0 H_c$, a multilayer structure with alternating in-plane uniaxial anisotropies is studied. This particular consideration is introduced to simplify the calculation of the equilibrium angles, as in the case of multilayers with similar anisotropies, determining the equilibrium ground state at $\mu_0 H_0 > \mu_0 H_c$ becomes challenging. Besides, as shown in Figs. 3.4(a)-(b), the band structures (evaluated at the critical fields) calculated for the cases $N = 30$ and 31 with alternating anisotropies show a behavior similar to those cases discussed previously. Indeed, the soft modes show a surface nature for the odd case (Fig. 3.4(c)) and bulk behavior for the even number of layers (Fig. 3.4(e))

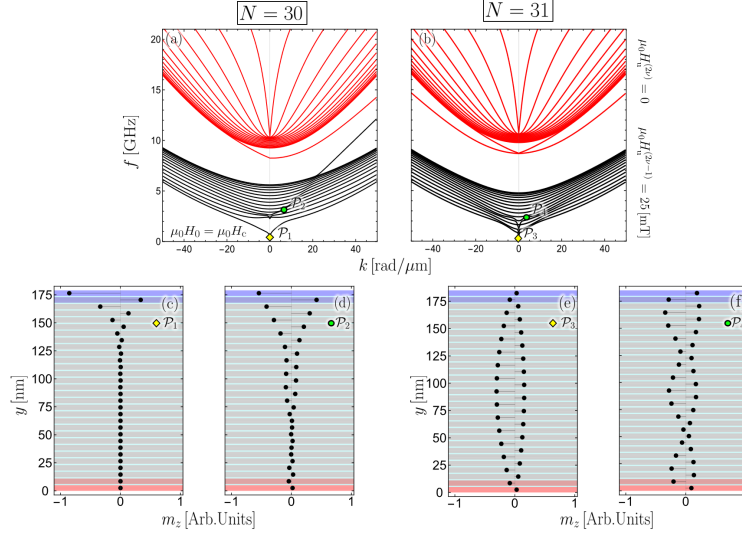


Figure 3.4: (a) and (b) show the SW band structure with the anisotropy values $\mu_0 H_u^{(2\nu-1)} = 25$ mT for odd layers and $\mu_0 H_u^{(2\nu)} = 0$ for even layers for $N = 30$ and $N = 31$, respectively. The external magnetic field is evaluated at 16 mT (a) and 26 mT (b), corresponding to the critical fields $\mu_0 H_c$. (c) and (d) illustrated the SW dynamic profiles m_z along the thickness for the case $N = 30$, and (e) and (f) for the system with $N = 31$.

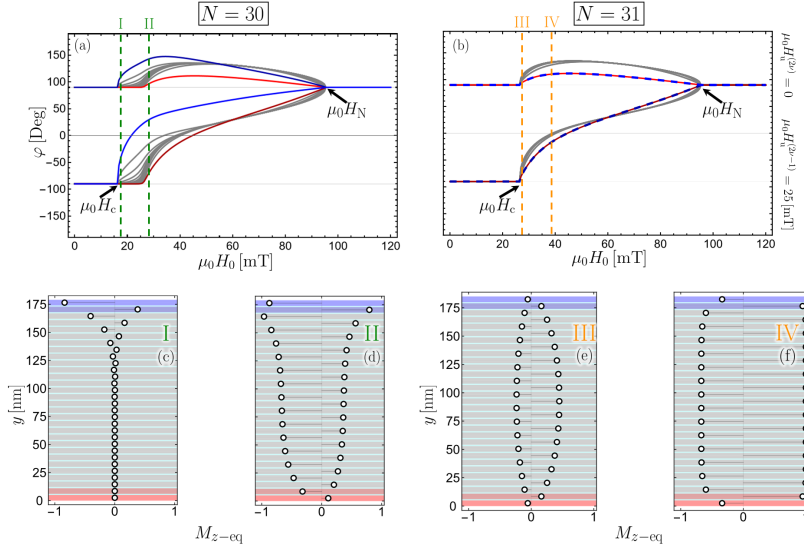


Figure 3.5: Equilibrium angles calculated for $N = 30$ and $N = 31$ in (a) and (b), respectively. The top and bottom equilibrium angles are highlighted in blue and red, respectively. (c) and (d) show the normalized z -components of the equilibrium magnetization ($M_{z-\text{eq}}$) evaluated at $\mu_0 H_0 = 18$ mT (I) and 28 mT (II), respectively, for the case $N = 30$ and (e) and (f) illustrated $M_{z-\text{eq}}$ at $\mu_0 H_0 = 27$ mT (III) and 38 mT (IV), respectively, for the system with $N = 31$.

The equilibrium angles are determined by numerically solving the system of equations 2.41 as a function of the external field. As mentioned before, to facilitate calculation, the anisotropy values used are $\mu_0 H_u^{(2\nu-1)} = 25$ mT for odd layers and $\mu_0 H_u^{(2\nu)} = 0$ for even layers. Figure 3.5 illustrates the equilibrium angles for scenarios (a) $N = 30$ and (b) $N = 31$, where phase transitions are indicated by the critical field $\mu_0 H_c$ and the nucleation field $\mu_0 H_N$. These fields delineate the transitions between antiparallel to spin-flop regimes, and from spin-flop to ferromagnetic phases, respectively. Figure 3.5(a) presents the equilibrium angles as a function of the applied magnetic field $\mu_0 H_0$ for the case with $N = 30$. Initially, the system is in an antiferromagnetic configuration, where the magnetization directions alternate between $\pm\pi/2$. As the field increases, a non-uniform behavior is observed along the thickness of the structure. Close to the critical field, it is observed that the layers located near the top of the structure (depicted in blue tones) begin to reorient before the others. This behavior can be attributable to the fact that the upper layer is initially oriented in the opposite direction to the external field, which causes the reorientation process to initiate at this surface. With a further increase in the field amplitude, the layers located in the bulk (shown in gray) begin to reorient, following the upper edge layers, until they reach the lower layers (represented by red tones). When the field approaches the nucleation field, approximately 92 mT, all the layers begin to collectively reorient, entering the ferromagnetic state.

On the other hand, figure 3.5(b) depicts the equilibrium angles for the case with $N = 31$. In the critical field, all layers initiate the reorientation process simultaneously. However, the layers located in the bulk exhibit a larger angular deviation compared to those at the top and bottom outer layers. This difference in amplitude reflects greater stability at the boundaries, which can be attributed to a contribution of the effective anisotropy and the fact that the top and bottom layers are parallel to the applied field.

As illustrated in Figures 3.5(c)-(f), where the z -component of the equilibrium magnetization $M_{z-\text{eq}} = \cos(\varphi)$ is calculated as a function of the external magnetic field $\mu_0 H_0$, the magnetic textures formed close the the critical fields are quite similar to the spin-wave profiles illustrated in Figs. 3.4(c) and 3.4(e). Additionally, the critical fields are identical to those obtained from the SW dispersions. This agreement between the z -component of the equilibrium magnetization and the dynamical profiles confirms the connection between the surface and bulk modes in the band structures on the onset of the magnetization reorientation process. In what follows, this idea is explored in more complex multilayer structures, where the influence of a perpendicular anisotropy is considered.

3.2 Stripe-like domains in multilayers

In the next part of the study, structures with ferromagnetic layers of significant thickness, comparable to or exceeding the magnetic exchange length, are explored. Additionally, the interfacial nature of the perpendicular anisotropy and interlayer exchange is also considered. In particular, the idea is to study whether soft modes can reach zero frequency at finite wave vectors, $k \neq 0$, suggesting the formation of magnetic textures, such as helices or stripe-like domains. The magnetic parameters used for these structures are the same as those in the previous section 3.1. However, now a perpendicular anisotropy is considered as mentioned above. This interaction is included because it is typical of this kind of interfacial energy when a ferromagnet is coupled with a non-magnetic material, like Ruthenium or Platinum. Furthermore, as shown in the following sections, such an interaction is crucial for the formation of stripe-like domains, which will be analyzed in this section.

The introduction of a perpendicular anisotropy field ($\mu_0 H_s$) leads to a distinctive behavior of the soft modes under certain conditions. In figure 3.6(a), a phase diagram is depicted as a function of both the external magnetic field and the anisotropy field. This diagram corresponds to a multilayer system composed of $N = 10$ layers, each one with a thickness of $d_\nu = 2$ nm. For perpendicular anisotropy fields below 800 mT, the system can transition from the parallel to the antiparallel state via the spin-flop regime. Starting from the saturated case at high fields, this transition initiates at a nucleation field $\mu_0 H_N$ and concludes at a critical field $\mu_0 H_c$. As observed in the SW band structures in Figures 3.6(b) and 3.6(e), the non-alignment state is expected to be a spin-flop regime. This can be anticipated through the soft modes since they exhibit the minimum frequency at $k = 0$. Therefore, a behavior similar to the one presented in Section 3.1 is obtained. In this range, both the nucleation and the critical fields are independent of the anisotropy field.

For the parallel state, around $\mu_0 H_s = 800$ mT, the frequency spectrum exhibits a distinctive mode characterized by three minima located at $k = 0$ and $\pm k_c$ (see Fig. 3.6(f)). This feature suggests that the instability of the parallel state can be initiated by a spin flop process or through the formation of a periodic spin texture with a characteristic periodicity $\lambda_c = 2\pi/|k_c|$, which corresponds to the emergence of stripe-like domain structures [28]. Then, as $\mu_0 H_s$ further increases, the frequency minima are characterized by non-zero wave vectors, as shown in Fig. 3.6(g). At this point, the ferromagnetic state is no longer stable, and stripe domains emerge. In contrast, the destabilization of the antiparallel state near to $\mu_0 H_s = 800$ mT (see point $\mathcal{P}_2$) is characterized by a dispersion showing two minima at $k = 0$ and at a negative wave vector $k_c < 0$ (see Figure 3.6(c)). This suggests that the instability can start through a spin-flop process or the formation of a chiral stripe.

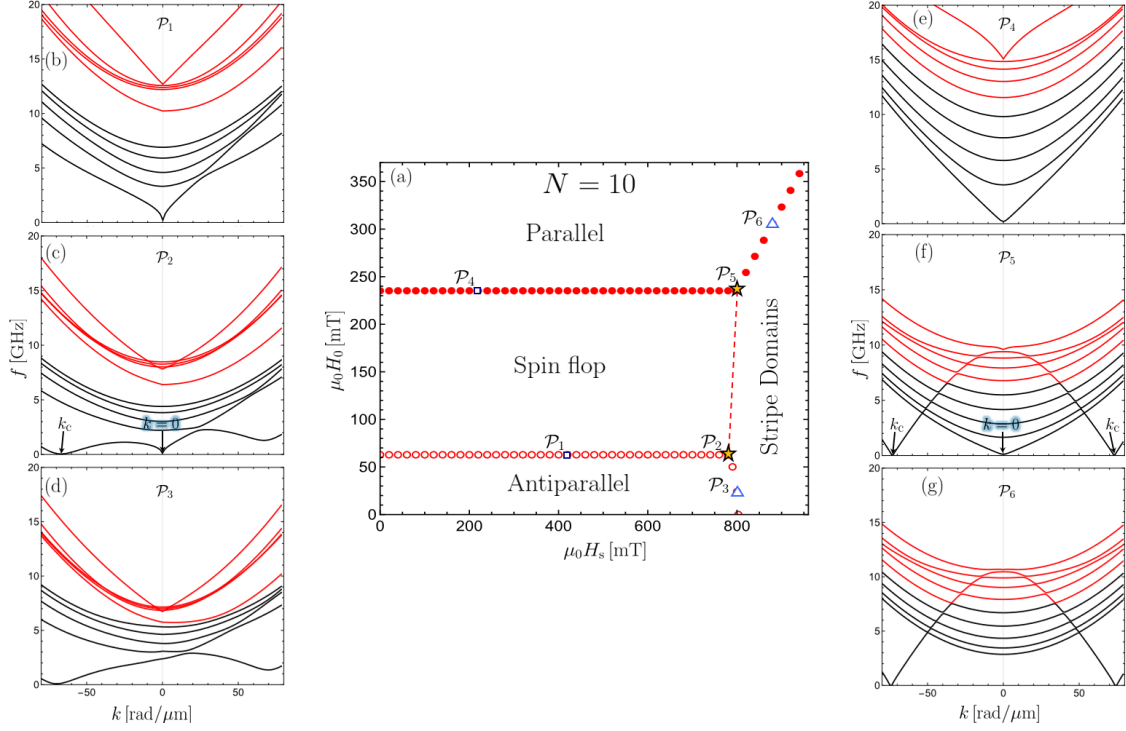


Figure 3.6: Magnetic phase diagram illustrating the transitions between antiparallel, parallel, spin-flop and stripe domains states as a function of the external magnetic field $\mu_0 H_0$ and the perpendicular $\mu_0 H_s$, for a multilayer system composed of $N = 10$ layers, each with a thickness of $d_\nu = 2$ nm. Solid dots represent the parallel state, whereas the open dots correspond to the antiparallel state. Panels (b)-(g) display the band structures associated with different states indicated in the diagram.

The representation of soft-mode profiles is better appreciated by examining the figure 3.7. Here, the arrows indicate the direction of magnetization, with blue and red colors representing orientations along $-x$ and $+x$, respectively, which correspond to the two minima of the state $\mathcal{P}_2$ (see Figure 3.6(c)). In Fig. 3.7(a), associated with the oscillation mode $k = 0$, the magnetization destabilization is observed at the top outer layer. This pattern persists along the $x - z$ plane, which can be understood as the limit case $\lambda_c \rightarrow \infty$ (spin-flop phase). In contrast, for the mode at $k = k_c$, Fig. 3.7(b) illustrates a “stripe domain” pattern, reflecting the system’s periodic structure when instability arises, pointing to a more complex phase. It is observed that the magnetization in the different layers along the z axis exhibits a phase shift. Namely, in one layer, a periodic up-down-up pattern is formed, while in the adjacent layers, the inverted pattern appears [28]. This alternating arrangement of magnetization is characterized by a length λ_c , which defines the spatial period of the texture.

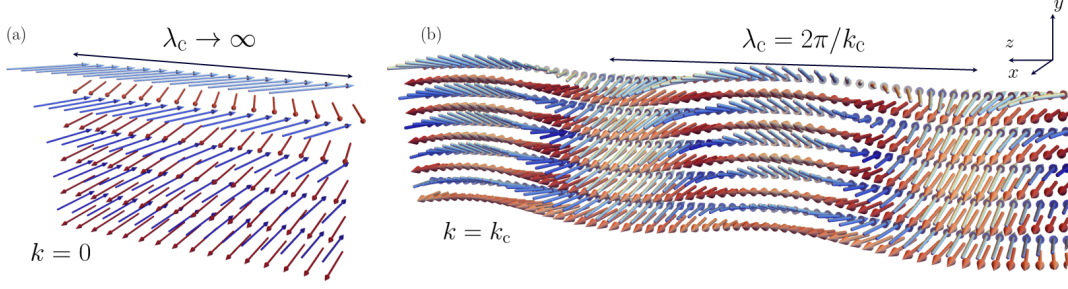


Figure 3.7: Representation of the oscillation modes calculated at the frequency minima of figure 3.6(c), where in (a) $k = 0$ and (b) $k = k_c$.

Figure 3.8 shows the representation of the oscillation modes at the minima for $k = 0$ and $k = k_c < 0$ corresponding to the $\mathcal{P}_5$ state (see Fig. 3.10(f)). In this case, figure 3.8(a) represents the oscillation mode for $k = 0$, where it is observed that the magnetization evolves predominantly in the volume of the system along the z axis, indicating a uniform bulk mode that will be related to the spin-flop phase. On the other hand, Fig. 3.8(b) illustrates the periodic magnetic texture formed along the z axis when the parallel state becomes unstable. Unlike the case described above, this magnetic texture does not exhibit a phase shift between adjacent layers; instead, all layers are in phase, forming a pattern of up-down-up magnetizations.

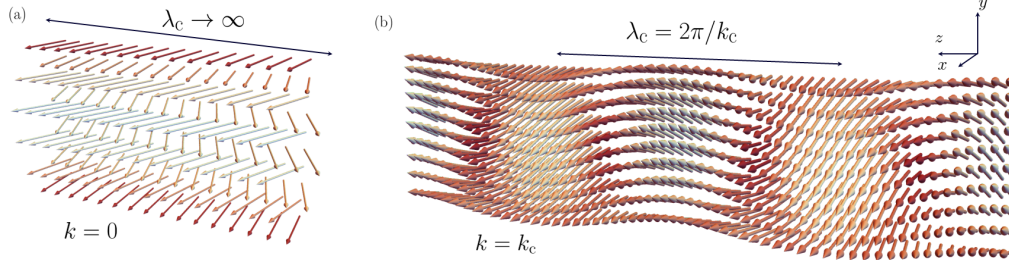


Figure 3.8: Representation of the oscillation modes associated with the minima of Figure 3.6(f) in (a) $k = 0$ and (b) $k = k_c$.

For the phase diagram shown in figure 3.6(a), the points $\mathcal{P}_2$ and $\mathcal{P}_5$, marked with a star, represent the minimum value of the anisotropy field required for the formation of the stripe domains in the antiparallel and parallel states, respectively. The behavior of these states is now analyzed as a function of the number of layers. Specifically, Figs. 3.9(a)-(b) display the minimum perpendicular anisotropy field required to induce these textures, while Figs. 3.9(c) and (d) present the critical and nucleation fields ($\mu_0 H_c$ and $\mu_0 H_N$), respectively. Furthermore, figures 3.9(e) and (f) show the characteristic length associated with each state. Red and orange dots denote antiparallel states with an odd and even number of layers, respectively. Blue and open squares identify parallel states with odd and even numbers of layers, respectively.

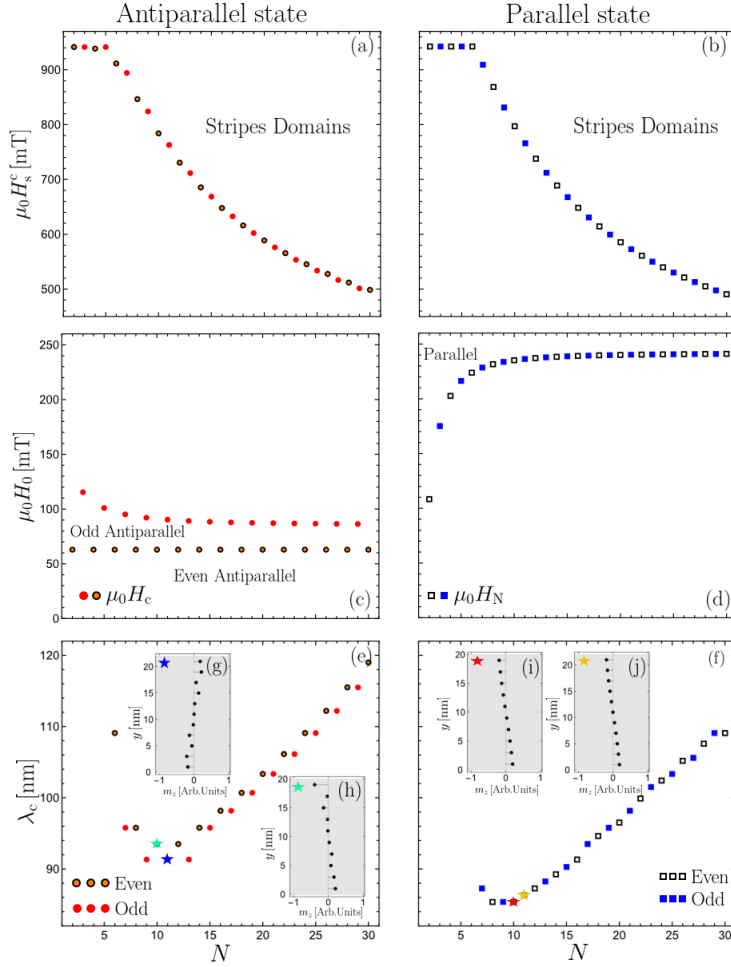


Figure 3.9: (a)-(b) Minimum anisotropy field $\mu_0 H_s^c$ required for the formation of stripe domains. (c)-(d) critical field (red and orange dots) and nucleation field (blue and open squares); and (e)-(f) characteristic length of the stripe domains as a function of the number of layers N . (g)-(j) illustrated the SW dynamic profiles for the case $N = 10$ and 11.

As shown in Fig. 3.9(a) and 3.9(b), the minimum anisotropy field $\mu_0 H_s^c$ (critical perpendicular anisotropy field) decreases as N increases. Regions above these curves correspond to the stability of stripe domain states, while those below indicate magnetic states compatible with the spin-flop transitions. In both the antiparallel and parallel states, the curves exhibit similar behavior, and the trend is similar for both even and odd numbers of layers. This behavior can be understood by considering the dipolar interaction. As the system increases its thickness (by increasing N), the dipolar interaction also increases, and hence these domains are more likely to form as N increases, requiring thus a smaller anisotropy to be formed.

For the critical field, a distinct contrast is observed between the even and odd cases as shown in Fig. 3.9(c)-(d). Specifically, the critical field is independent of the number of layers for even-layered systems, whereas it decreases for odd-layered systems [27, 29].

This disparity is due to the different symmetries presented by these systems. For the even case, there is always at least one layer pointing in the opposite direction of the external field, and therefore, there will be no net magnetization. Consequently, it is expected that the field necessary to destabilize the system will be smaller. On the other hand, for the odd case, the opposite occurs; there is a net magnetization, so a larger field is required to overcome the coupling between the layers. In the case of the nucleation field, figure 3.9(d) shows that there is no difference between even and odd numbers of layers [28]. Besides, as the number of layers increases, the nucleation field tends to a constant value. This behavior is related to the bulk nature of the ferromagnetic state's destabilization. In this case, there is a point at which incorporating more and more layers at the top or bottom does not significantly alter the nature of the reversion, which begins at the center of the multilayer structure.

Figures 3.9(e)-(f) illustrate the variation in the characteristic length or periodicity (λ_c) of the stripe domains as a function of the number of layers for both antiparallel and parallel states. In both cases, there is a non-monotonic behavior of the periodicity. For instance, in the parallel state, stripe-like domains begin to form at $N \geq 7$, where the length initially decreases and then increases with the addition of further layers. The behavior of the curve can be interpreted by analyzing the main energy contributions associated with the domains. For thin structures, the dipolar interactions that cause the periodic domains are not enough to compete with the exchange interaction that promotes a spin texture with $\lambda_c \rightarrow \infty$. Then, as the thickness of the multilayer increases (by incorporating more layers), the dipolar interaction is enhanced, and stripe domains are formed with a periodicity that decreases as N increases. Because the minimum anisotropy field required to form the stripe ($\mu_0 H_s^c$) starts to decrease (see Figs. 3.9(a) and 3.9(b)), the characteristic length increases again. It is interesting to note that, in both cases of parallel and antiparallel destabilization, the periodicity of the spin texture has a similar dependence on the number of layers. Nevertheless, for the antiparallel case shown in Fig. 3.9(e), there is a difference between the odd and even number of layers. This is related to the fact that the domains in this case are sensitive to the outer layers, producing chiral domains for even N and non-chiral domains for odd N . In the profile shown in Fig. 3.9(h), it is noted how the amplitude of the magnetization is notable at the top layer, while in the other cases (Figs. 3.9(g), (i) and (j)) the outer layers do not show a prominent SW amplitude in comparison with the rest of the layers.

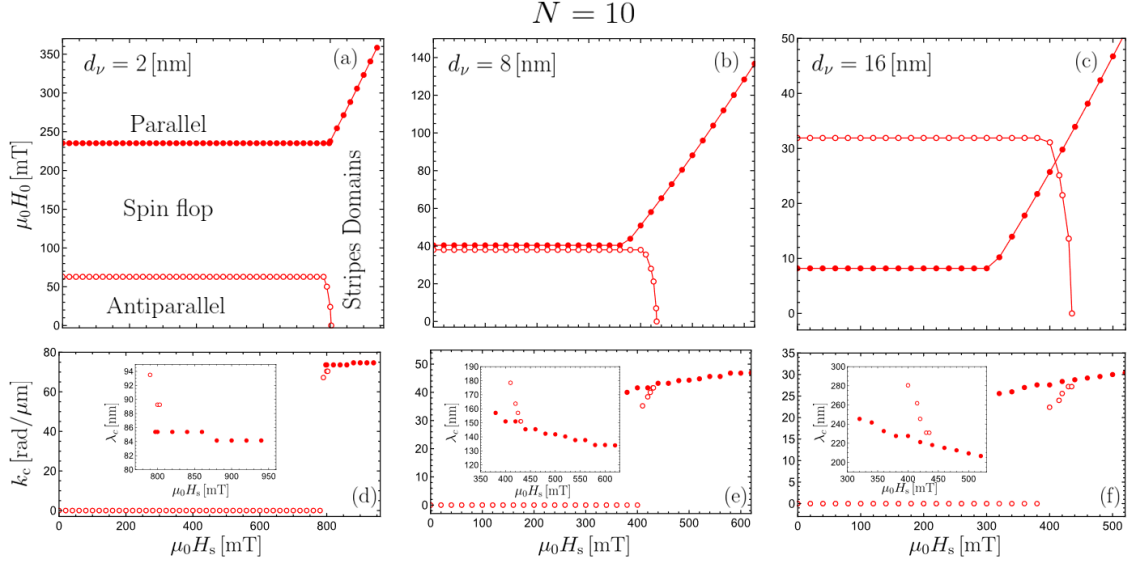


Figure 3.10: (a)-(c) Phase diagrams for an antiferromagnetic coupled multilayer system, composed of $N = 10$ layers with thicknesses $d_\nu = 2$, 8, and 16 nm, respectively. (d)-(f) critical wave vector, and (g)-(i) characteristic length, both as a function of the perpendicular anisotropy field.

Finally, phase diagrams for systems with a significant thickness are analyzed. These diagrams are illustrated in figure 3.10 for the case $N = 10$. As the thickness increases, the spin-flop phase disappears, as seen in Figs. 3.10(b) and 3.10(c). In the latter case, bistable states are identified, meaning that both the antiparallel and parallel states are stable within a certain range of the external field. Furthermore, stripe domain states become increasingly accessible because the dipolar interaction becomes more significant with increasing thickness, favoring the formation of magnetic stripe-like textures. Likewise, the perpendicular anisotropy field required to stabilize these textures decreases as the layer thickness increases. This behavior is corroborated in Figs. 3.10(d)-(f), which display the critical wave vector k_c as a function of the anisotropy field. The insets of figures 3.10(d)-(f) evidence the dependence of the periodicity on the perpendicular anisotropy field $\mu_0 H_s$. Overall, λ_c decreases with the increase of the anisotropy strength, being the increase ratio more prominent for the stripe domains formed from the antiparallel destabilization.

Chapter 4

Conclusions

In this thesis, the formation of spin textures in magnetic multilayers has been investigated through the analysis of spin-wave dynamics. In particular, soft modes were employed to predict the emergence of stripe-like domains in multilayered systems with perpendicular magnetic anisotropy. It was found that the formation of such domains requires a critical perpendicular anisotropy field, which differs depending on whether the destabilization originates from the antiparallel or parallel alignment. Near the nucleation field, chiral spin textures are predicted to form, whereas in the vicinity of the critical field associated with antiparallel destabilization, chiral stripe-like domain structures are expected. This distinction is attributed to the bulk or surface character of the corresponding magnetic textures. Moreover, it was observed that increasing the number of layers leads to a reduction in the minimum anisotropy field required to stabilize periodic spin textures. This effect is attributed to enhanced dipolar interactions that promote stripe formation as the system becomes thicker. Similarly, for a fixed number of layers, increasing the individual thickness of the ferromagnetic layers also reduces the critical anisotropy field needed for stripe formation, again due to the strengthening of dipolar interactions with increasing film thickness. In conclusion, the analysis of soft modes in multilayer systems proves to be a powerful and efficient approach for predicting the emergence of periodic spin-textured domain structures, offering a practical alternative to the demanding and often intractable task of calculating the corresponding static configurations through analytical methods.

As a natural extension of this thesis, further analysis of graded multilayer systems is particularly interesting. In such systems, parameters like the saturation magnetization can be varied along the thickness to tailor the formation of chiral stripe-like domains. Incorporating chiral interactions, such as the Dzyaloshinskii–Moriya interaction, would allow for a detailed investigation of their impact on the periodicity and chirality of these domain structures. Additionally, exploring configurations with strong perpendicular anisotropy, where the magnetization lies entirely out of the plane, offers further insight into the stability and nature of the resulting textures. To enhance the validity and

predictive scope of the theoretical results, comparisons with micromagnetic simulations are also essential.

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