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“Identification of new physics signals based on deep
learning techniques in dark matter direct detection
experiments”

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Dedication

Dedicated to my mother Deonne Aceituno, my father Cristián Alvarado, and my little brothers, Benjamín and Tomás Alvarado; who are always supporting me in every aspect of my life.

Acknowledgment

I want to thank my best friends, Diego Ricci and Sebastián Ávalos, for accompanying me in the long journey that has been this work. I also want to thank to all the people that have helped me in the way and that is currently or was part of my life.

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Abstract

Abstract— The constant effort to develop better detectors in the search of Dark Matter (DM) has made them more sensitive to noise in the measurements making more difficult the task of event classification. Liquid xenon detectors arise as a promising way of finding DM. Leveraging advancements in Deep Learning (DL), this work presents a novel framework for classifying events from DM detectors under the Standard Model of Particle Physics (SM) and Beyond Standard Model (BSM) neutrino interactions. The proposed methodology combines simulations of detector interactions and Neural Network (NN) based classification to tackle the challenges posed by degeneracy caused by overlapping signals. This framework was tested using simulated data presenting degeneracy with the SM neutrinos considering Weakly Interacting Massive Particle (WIMP)s and neutrino vectorial Non-Standard Interactions (NSI), yielding high classification performance with mean Area Under the receiver operating characteristic Curve (AUC) scores approaching 1. Key contributions include the development of a novel event classification framework and insights into the parameter space regions of WIMPs and BSM neutrinos. These findings not only validate the feasibility of applying the Deep Support Vector Data Description (Deep-SVDD) model but also demonstrate its potential for a more detailed framework and its applications in the fields of DM and particle physics.

Keywords— WIMP, Dark Matter, Direct Detection, Deep Learning, Anomaly Detection

Acronyms

Symbols

Λ CDM

Λ Cold Dark Matter. 32, 34

ν

Neutrino. 26, 27, 45–51, 55–62

ν BD

Neutrino-capture Beta Decay. 31

pp chain

proton-proton chain. 28, 29

A

AD

Anomaly Detection. 1, 23, 51

AI

Artificial Intelligence. 10, 11

ANN

Artificial Neural Network. 15, 17, 23, 51

AUC

Area Under the receiver operating characteristic Curve. iv, 2, 25, 26, 60, 62

B

BSM

Beyond Standard Model. iv, 6, 9, 38, 41, 45–51, 55–63

C

CC

Charged Current. 29–31

CE ν NS

Coherent Elastic Neutrino-Nucleus Scattering. 27, 31

CMB

Cosmic Microwave Background. 32

CNN

Convolutional Neural Network. 20, 21, 51, 52, 62

CNO cycle

Carbon-Nitrogen-Oxygen cycle. 28–30

D**DD**

Direct Detection. 1, 3–9, 26, 33–36

Deep-SVDD

Deep Support Vector Data Description. iv, 1, 2, 23–25, 51, 58, 62

DL

Deep Learning. iv, 2, 5, 6, 8–11, 15, 62, 63

DM

Dark Matter. iv, 1–9, 26, 31–38, 41, 42, 47, 54, 55, 62, 63

DRIFT

Directional Recoil Identification from Tracks. 9

F**FPR**

False Positive Rate. 25

G**GC**

Galactic Center. 33, 35, 36

L**LUX**

Large Underground Xenon. 9, 37

LZ

LUX-ZEPLIN. 9, 38

M

ML

Machine Learning. 1, 5, 6, 8–13, 15, 22

N

NC

Neutral Current. 31

NGI

Neutrino Generalized Interactions. 41

NN

Neural Network. iv, 1, 9, 15, 17–19, 23, 24, 51, 54, 56–58, 63

NSI

Non-Standard Interactions. iv, 4, 6, 7, 9, 41

Q

QFT

Quantum Field Theory. 39

QM

Quantum Mechanics. 39

R

ReLU

Rectifier Linear Unit. 17

ROC

Receiver Operating Characteristic. 25

S

SD

Spin-Dependent. 35

SGD

Stochastic Gradient Descent. 19, 20

SI

Spin-Independent. 35

SM

Standard Model of Particle Physics. iv, 6, 7, 9, 34, 38–41, 46, 49, 51, 58–60, 62

SMBH

Super-Massive Black Hole. 32, 33

SSM

Standard Solar Model. 30, 41, 62

SSN

Standard Solar Neutrino. 46, 48–50, 62

T**TPC**

Time Projection Chamber. 37

TPR

True Positive Rate. 25

W**WIMP**

Weakly Interacting Massive Particle. iv, 4, 6, 34–36, 42, 45–51, 54–57, 59–62

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Chapter 1

Introduction

The search for Dark Matter (DM) represents one of the most intriguing challenges in modern physics, stemming from anomalies first observed in the behavior of galaxies and the cosmos. Newtonian mechanics, supplemented by Einstein's general theory of relativity, provided a framework for understanding celestial mechanics. However, as observational precision increased, phenomena such as the unexpected rotation curves of spiral galaxies and discrepancies in galactic mass distribution revealed the presence of unseen matter. The pioneering work of Jan Oort and others hinted at the existence of this enigmatic substance, now called **dark matter**, which interacts gravitationally but remains invisible due to the absence of electromagnetic interactions.

Among the proposed methods to detect DM, Direct Detection (DD) has garnered significant attention. This approach relies on sensitive detectors filled with specific materials to capture the elusive interactions between DM particles and atomic nuclei. However, as these detectors grow in scale and sensitivity, they inevitably encounter new challenges. The “neutrino floor”, a background signal arising from constant solar neutrino interactions, is one such obstacle. Solar neutrinos mimic the signals of DM interactions, complicating the differentiation between potential DM events and background noise. Addressing this issue demands not only advancements in detector technology but also innovative techniques to simulate, analyze, and interpret data.

In this context, scientific computing emerges as a critical ally, bridging the gap between physics and computational science. By creating detailed simulations of detector behavior and particle interactions, researchers can validate theoretical models, justify experimental designs, and optimize detector configurations before committing to the immense resources required for physical implementation. This interdisciplinary collaboration enables the exploration of DM detection in a manner that is both efficient and rigorous, paving the way for future breakthroughs in understanding the Universe's most elusive constituent.

This work is structured to guide the reader through the progression of this problem, its conceptualization, and its resolution. In **Chapter 2**, the context and problem definition are presented, highlighting the challenges of degeneracy and the imbalance between known signal distributions and hypothesized phenomena. Current solutions are searched, and the research goals and contributions are clearly defined. **Chapter 3** establishes the conceptual framework, delving into the relevant physics of DM, the role of neutrinos, and the technological foundations of Machine Learning (ML), including the Anomaly Detection (AD) framework employed in this work.

The **Proposed Solution** in **Chapter 4** outlines the methodology, starting with simulation configurations, dataset generation, and preprocessing techniques, before detailing the development of the Neural Network (NN) and its architecture. The design takes advantage of a Deep

Support Vector Data Description (Deep-SVDD) model to address the imbalance and degeneracy challenges, creating a framework that is both robust and adaptable for this class of problems.

In **Chapter 5**, the proposed solution is validated through rigorous testing, including an analysis of degeneracy, training the Deep-SVDD model, and evaluating its performance. Metrics such as Area Under the receiver operating characteristic Curve (AUC) are used to quantify the model's ability to distinguish between overlapping signal classes.

Finally, **Chapter 6** provides the conclusions drawn from this work, summarizing its contributions and highlighting areas for future development. These include enhancing simulations, exploring alternative architectures, and integrating experimental datasets to refine and expand the applicability of the framework.

This work not only demonstrates the feasibility of using Deep Learning (DL) for DM signal identification but also lays the groundwork for future studies, combining theoretical and phenomenological approaches to advance the field of astroparticle physics.

Chapter 2

Problem Definition

2.1 Context

Since Sir Isaac Newton published his “*Philosophiæ naturalis principia mathematica*”, several physicists and astronomers have been using his equations to describe the motion of bodies, both in Earth and in the Universe. With more precise observations of what is found beyond the solar system, and with the general theory of the relativity of Albert Einstein [Bertone et al., 2005], a lot of breakthroughs have been possible in the fields of physics and astrophysics. Jan Oort “was the first to perform an analysis of the vertical equilibrium of the stellar distribution in the solar neighborhood”, with this he concluded that “there is more mass in the galactic disk than can be accounted for by star counts” [Bosma, 1998]. This along with other prior observations, were the first anomalies discovered that lead to the consolidation of the idea that there is **Dark Matter** that we can't see.

There is a lot of reasons to believe in the existence of this matter that we can't see, some of them are: cosmological data, astrophysical observations, the formation of large-scale structures, galaxy rotation curves (specifically of spiral galaxies), among others. The biggest evidence of the DM is its own nature, namely its gravitational interaction and its absence of electromagnetic interaction. Some properties of the DM in our Galaxy (The Milky Way) can be initially derived from the observation of the rotation curves [Lisanti, 2016].

The particle and astroparticle physics community, together with experimental physicists, have proposed different methods for the detection of DM. They can be classified into three main categories: Direct Detection, indirect detection and direct production.

- Direct Detection: this method proposes the use of chambers filled with specific types of atoms to identify when a DM particle collides with one of its nuclei. The detectors inside the chamber should be able to measure the energy of the recoils, and with that infer the properties of the particle that collided with the nucleus given its kinematics. A signal above background will be an unquestionable proof of DM detection.
- Indirect detection: this method proposes the use of telescopes and satellites to search for the effects produced by the annihilation of DM particles. These instruments should be pointed to the center of our Galaxy and other areas where DM should accumulate given its gravitational effects. A signal above astrophysical background will establish an irrefutable detection of DM.
- Direct production: this method proposes the use of particle accelerators to produce DM as a by-product of a particle collision. Particles collide inside the accelerators, new particles show up as a result, and the detectors around the accelerator measure

the properties of these particles. A signal above collider background will imply DM detection.

In particular, the efforts using DD for the search of DM have caused some consequences: construction of underground experiments to eliminate the noise of cosmic rays, recoil studies of some atoms, development of detectors with tons of detector material (multi-ton scale detector) to increase sensitivity, search of new neutrino Non-Standard Interactions (NSI) to get better accuracy identifying the signals of DM, among several others. All these consequences are interrelated due to the constant effort to develop better DM detectors, increasing the interaction probability with the detector material using large amounts of it (tons). Being more sensitive means that they become more sensitive to noise in the measurements, detecting things that were previously ignored, signals of particles that may seem of interest but are not, for example. This is what happens with the “neutrino floor” (ν -floor) or solar neutrino background, that is going to appear as a constant source of noise as the detectors increase their sensitivity because the Sun is constantly radiating neutrinos over us, so it has become of interest to find out more things about the neutrinos.

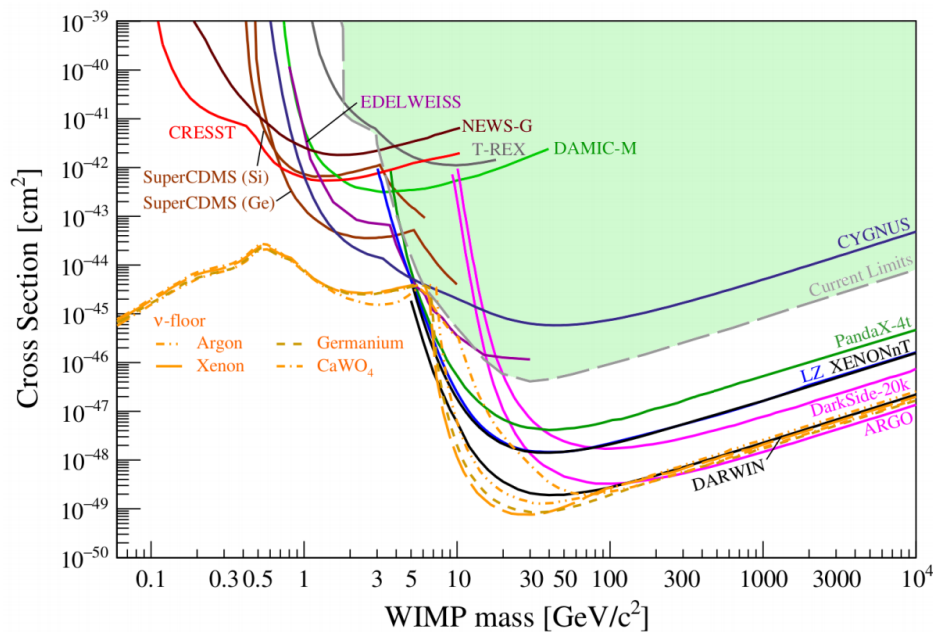


Figure 1: Sensitivity projections for spin-independent WIMP-nucleon scattering associated to the experiments planned and in development. Source: [Billard et al., 2021].

Many experiments of DD are being carried out to this day, and there are others that are being developed, always trying to increase the accuracy and sensitivity of the detectors. Almost all of them are trying to detect a kind of particle that is very likely to have the same nature as the DM, the Weakly Interacting Massive Particles (WIMPs). Figure 1 shows the projected sensitivities of DARWIN, it also shows the neutrino floor in different kinds of yellow lines depending on the detector materials used in the different experiments. The green area in

Figure 1 shows the area currently covered by experiments and detectors taking data. As suggested by the figure, one goal of all these experiments is to increase the current range of detection, meaning that they need to get near the limit of the neutrino floor.

These types of experiments involve a difficult task, not only because of the great challenge that their development entails, but also because of the financing necessary to carry them out. It's for this reason that simulations of the experiment are necessary, to justify that the physics and the theoretical detector should work. Here is where physics joins forces with computer science.

Scientific computing applications is an area of computer science that deals with the scientific requirements to carry out experiments. Specifically, it uses algorithms and models to assist in the development of simulations and in the evaluation of its results. Some situations where the help of scientific computing is needed are when the experiments are big in scale, dangerous, and/or expensive. Thus, scientific computing is a multidisciplinary field where experts from the field that requires the modeling and experts that can develop the algorithm join forces [Team, 2021].

2.2 Problem Description

Computer science, particularly through the advancements in ML and DL, has provided powerful tools for tackling data-driven challenges. These methodologies excel in scenarios involving large volumes of data, offering sophisticated techniques to analyze, interpret, and derive insights with exceptional efficiency and accuracy. G. Carleo et al. mention that “in parallel to the rise of ML techniques in industrial applications, scientist have increasingly become interested in the potential of ML for fundamental research, and physics is no exception. To some extent, this is not too surprising since both ML and physics share some of their methods as well as goals” [Carleo et al., 2019].

A very common problem in physics is to determine whether an event is of interest or not, and since the experiments and its detectors capture large amounts of data all the time, the task of manual labeling or tagging is virtually impossible. Thus, several techniques have been developed and implemented to address this problem, including techniques derived from ML.

The DD of DM also has this problem, and depending on the type of detector, different kind of data is retrieved as well as physical quantities, moreover, depending on the particular detector that is being used, different details about the sensitivity and expected noise have to be taken into account for any analysis.

Due to the great work that implies to work in the event analysis of one detector, great enough to be a PhD thesis, this work will address the problem of the classification of events from DM detectors in a phenomenological way, meaning, working with variables associated with the nuclear recoil that can be translated to the specific variables taking part in any particular

detector. A data generator provided by the expert supervising this work will be used too to emulate the data captured by a generic detector, phenomenologically¹. This work aims to contribute to the field of the DD of DM from the computer science point of view, applying DL techniques.

It is also worth to mention and clarify some of the concepts that have been used until now and will be used during this work:

- *Event*: the measurement of the physical quantities after the occurrence of a fundamental interaction between a particle and the atom nucleus of the detector material. This event would be any situation where happens a nucleus recoil.
- *Signal*: event of interest from the physical point of view. In this work, the signal would be an event where a particle that fits the characteristics of the nature of the DM, trigger a nucleus recoil.
- *Background*: also called “noise”, these are other kind of events that are not of interest and other events that could mimic the signal. In this case, the background could contain events produced by other particles such as neutrinos.
- *Neutrino Floor*: is the background noise expected by the solar neutrinos that are crossing the Earth all the time. Since the Sun can’t be turned off, this noise should always be present in the sensitivity analysis of the detectors.

Given that the neutrino background is permanently present, another problem arises. The current theoretical model, the Standard Model of Particle Physics (SM), describes the behavior of the fundamental particles including the neutrino, but it is not a complete nor a finished model, and it could lack some interactions and other particles. Consequently, the physicists have begun to search for neutrino NSI since the noise of the solar neutrinos of these detectors could hide new interactions. Some theories of new physics, physics Beyond Standard Model (BSM), explain these new interactions: massive vector interactions, light vector interactions, scalar interactions, among others [Bhupal Dev et al., 2019].

The other side of this problem comes when we consider that new physics signals could appear. In [Schneck et al., 2015], they mention that the use of ML techniques and likelihood analyses require accurate models of both the signal and the expected background, and the efforts of the modeling should concentrate not only in the background because “if a WIMP signal that does not conform to the standard spin-independent assumptions is present in the data, it could produce unknown effects on the final result because it may not match either the signal or the background model”. Thus, this work will also address the problem of the classification of events from DM detectors assuming neutrino NSI.

Another challenge in this work is the presence of signal degeneracies. This occurs when two or more phenomena generate indistinguishable distributions of events for a given parameter

¹The data generator is the one used in the paper by Aristizabal et al. [Aristizabal Sierra et al., 2019].

of interest—in this case, the recoil energy. As a result, it becomes difficult to discern which phenomenon is responsible for the observed signal. The extent of this problem also depends on the detector's sensitivity, as it determines the bin size of the histogram used to represent the event distribution.

2.3 Current Solutions

The focus of this work is the classification of events from DM detectors, considering both SM neutrino interactions and NSI. This particular perspective has not been explored previously. While efforts to identify and classify SM neutrino signals as noise in DM experiments are common, the scenario involving neutrino NSI has remained unexamined due to its inherent complexity. The introduction of two signals with the potential of being indicative of new physics results in an exponentially large number of parameter combinations to account for. Furthermore, determining whether a specific combination is allowed within the theoretical framework adds an additional layer of difficulty to the problem.

The event analysis of a detector is not a small work to do, so the solutions that exists depends on the experiment or collaboration in charge of building the detector. This is due to the different conditions, for example, the detection material used inside the chamber, the material used for the chamber, different sources of measurement error or uncertainty, the sensitivity, among others.

2.4 Goals

2.4.1 Main Goal

The main goal of this project is to identify Dark Matter signals in liquid xenon detectors, separating them from events associated to the solar neutrino noise, including possible new physics signals in the background.

2.4.2 Specific goals

1. Form a base knowledge about the physics involved in the Dark Matter problem and its Direct Detection.
2. Develop an algorithm that allows the identification of Dark Matter in a liquid xenon detector.
3. Tune the algorithm to identify the neutrino noise signals assuming standard and non-standard theory.

4. Present a new technique for the identification of Dark Matter signals in experiments that use tons of detector material.

2.5 Contribution

This work offers a first step into the path of the development of a generic tool that would help the particle and astroparticle physics community, together with experimental physicists, and the community of DD of DM.

Almost all the researches and developments that are currently being held, works with the physical quantities retrieved from the DM detectors, so working in a phenomenological way offers a generic framework to begin the development of a tool that could classify different kind of events associated to the DD of DM.

Then, this is the first work that uses an algorithm of ML, or more specifically, DL to classify the events of a DM detector in a more generic way (because its phenomenological nature) compared to the existing specific solutions, thus allowing to be adapted and used in real experiments in the future.

This is a relevant matter because there is a lot of collaborations that are doing research to build the next generation of DM detectors, these are already being planned or in development as it can be seen in Figure 1. So, that is why a framework that can be used for all of them is of interest.

Chapter 3

Conceptual Framework

3.1 State of the art

Before the state of the art can be introduced, it's necessary to specify the characterization of the problem. The problem is the classification of events from DM detectors assuming SM interactions or neutrino NSI.

The ML comprises a diverse set of algorithms and modeling techniques used for a wide range of data processing tasks. Alongside the growing use of ML in industrial applications, researchers have increasingly recognized its potential for advancing fundamental scientific research, including in physics. This overlap is not entirely unexpected, as ML and physics share common methodologies and objectives. Both disciplines focus on collecting and analyzing data to develop models that predict the behavior of complex systems. Physics subfields that are already taking advantage of the benefits of ML include: particle physics, cosmology, quantum many-body physics, quantum computing, and chemical and materials science. In “Machine learning and the physical sciences”, Carleo et al. make a review showing “the recent research on the interface between ML and the physical sciences” [Carleo et al., 2019].

In particle physics, DL was used for the first time in a project that targeted hypothesized particles from BSM [Baldi et al., 2014] where the NN was trained for an event-selection task over simulated data, showing an exceptional performance. In the field of DD of DM, applications of ML have already begun to be implemented for event discrimination in experiments like CYGNUS [Lynch, 2019], Large Underground Xenon (LUX) [Akerib et al., 2022], LUX-ZEPLIN (LZ) [Jahangir, 2022], XENONnT [Khosa et al., 2020] [Herrero-Garcia et al., 2022], and Directional Recoil Identification from Tracks (DRIFT) [Battat et al., 2021].

Although the current research on DD of DM considers different backgrounds that are characteristic of each experiment, solar neutrino background is always present, and this background could hide the signals of BSM physics as is explored in [Aristizabal Sierra et al., 2019]. In the same vein, a review [O'Hare et al., 2022] presented in the 2021 Snowmass Process² suggested the use of ML for the first time in a paper that addressed DM and BSM neutrinos.

The identification of DM signals in DD experiments has increasingly benefited from the application of ML under the assumption of standard interactions based on the SM. However, to date, no research has specifically focused on identifying potential DM signals in the context of neutrino NSI while using ML.

²<https://www.slac.stanford.edu/econf/C210711/>

3.2 Machine Learning

In this section the main sources of information are the books “Machine Learning: A Probabilistic Perspective” by Kevin P. Murphy [Murphy, 2012], “Deep Learning” by Ian Goodfellow, Yoshua Bengio, and Aaron Courville [Goodfellow et al., 2016], and “Geometry of Deep Learning: A Signal Processing Perspective” by Jong Chul Ye [Ye, 2022]. Any other source used will be cited accordingly.

In recent years, ML has become a transformative technology across different fields, including healthcare, finance, science, and engineering. Its ability to process large amounts of data and uncover hidden patterns has made it a cornerstone of modern Artificial Intelligence (AI). However, understanding ML requires a fundamental shift from traditional programming paradigms.

3.2.1 What is Machine Learning?

In 1959, Arthur Samuel defined ML as a “field of study that gives computers the ability to learn without being explicitly programmed” [Samuel, 1959]. Today, we can define ML more specifically as a “set of methods that can automatically detect patterns in data, and then use the uncovered patterns to predict future data or to perform other kind of decision-making under uncertainty” [Murphy, 2012]. One thing in common between these two definitions is the ability to discover, which means that without explicit programming, patterns and relationships can be discovered or learned (in data).

Although ML has been defined, it is important to state that it is different from DL and AI since these three concepts tend to be used interchangeably. DL is a subfield of ML, and ML is a subfield of AI, as Figure 2 shows.

AI is a field focused on creating machines and provide them with “intelligence”. Although there are several definitions for AI³, our primary focus lies in understanding the difference between AI and ML. Traditional AI systems are often built using classical programming methods, where the programmer explicitly defines the rules or constraints that govern the system’s behavior. In this approach, we provide both the rules and the data necessary for the algorithm to perform its computations, and in return, we obtain the desired output. ML, on the other hand, reverses this process. Instead of explicitly programming the rules, we supply the algorithm with data and, in some cases, the expected outputs. The algorithm then learns the underlying patterns or rules from the data itself, enabling it to generalize and make predictions for new inputs.

DL is a family of methods⁴ whose main characteristic is the use of fundamental units called

³For the interested reader, a rigorous definition of AI can be found in section “What is AI?” in the introduction of the book of Russell and Norvig [Russell and Norvig, 2010].

⁴DL as a subfield of ML follows the same paradigm where no explicit rules are stated, and only data is taken.

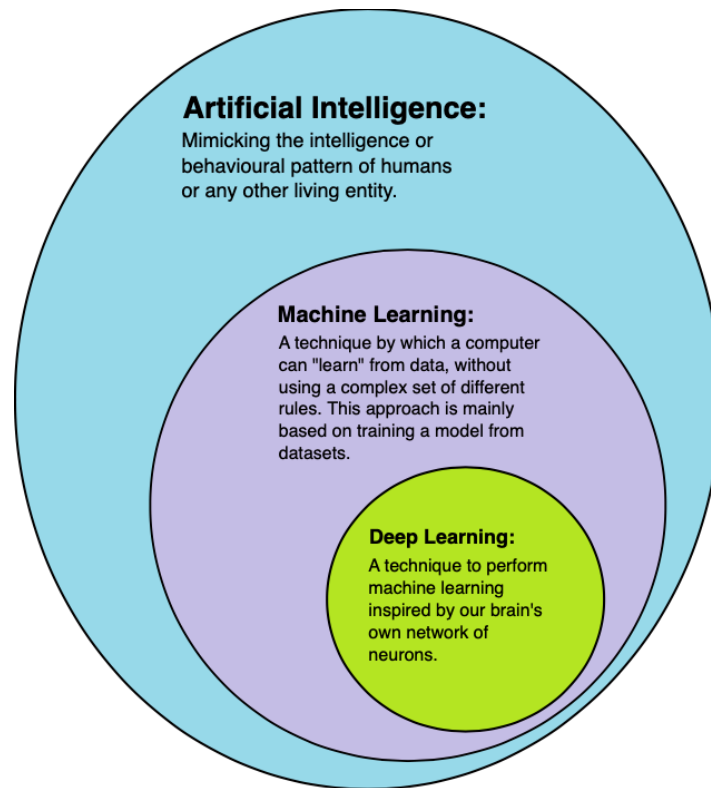


Figure 2: A Venn-diagram of Artificial Intelligence showing how Deep Learning is a subset of Machine Learning and how Machine Learning is a subset of Artificial Intelligence. Source: <https://commons.wikimedia.org/wiki/File:AI-ML-DL.svg>

neurons which are arranged in *layers*. Fundamentally, DL is so named because it relies on a deep hierarchy of concepts, where each layer of the model builds on simpler concepts to form more complex ones. If we visualize this process, the structure appears as a multi-layered graph with great depth, where each layer represents a different level of abstraction.

A *dataset* is a collection of examples or instances that the model uses to “learn” from. Each example typically consists of input data and, in supervised learning, a corresponding output or label⁵. The dataset provides the raw information from which the model extracts patterns, enabling it to make predictions or decisions on new, unseen data. The quality and quantity of the dataset are critical, as they directly influence how well the model generalizes.

If we recall the definition of ML, it says that it is a “set of methods”, so we can think of DL as a subset of these methods.

⁵Supervised learning, along with other types of learning, will be defined in the following subsection.

3.2.2 Types of Machine Learning

The ML can be categorized into three main types, depending on the nature of the data available and the specific goal of the task. Among these, supervised and unsupervised learning are the most commonly used and widely taught.

The first type is supervised learning, also known as predictive learning. In supervised learning, the goal is to learn the mapping from input data to output labels based on a set of labeled examples. The algorithm identifies the underlying rules that relate the input features, also referred to as attributes or covariates, to the corresponding outputs, also called the response variable. Inputs can range from simple numerical arrays to complex structures like images or text, while outputs are typically either categorical (e.g., class labels from a finite set) or continuous (e.g., real-valued predictions). The trained model can then generalize to new, unseen data, making it suitable for tasks like classification and regression.

The second type is unsupervised learning, also referred to as descriptive learning. Unlike supervised learning, it works with unlabeled data, aiming to uncover hidden patterns, structures, or relationships within it. The absence of labeled outputs means the algorithm focuses on grouping similar data points (clustering), reducing data dimensions, or identifying associations. This process is often described as knowledge discovery, as it seeks to reveal insights about the underlying data distribution.

The third type, reinforcement learning, is fundamentally different from the previous two. Rather than working with a fixed dataset, reinforcement learning involves an autonomous agent that interacts with an environment to learn how to perform a task. The agent learns through a process of trial and error, guided by feedback in the form of rewards or penalties. This feedback loop enables the agent to improve its actions over time without requiring human intervention, making reinforcement learning ideal for dynamic and adaptive systems like robotics and game playing.

3.2.3 Evaluating Models

Model Definition

A ML model can be viewed as a mathematical function that maps inputs to outputs. This function is parameterized by a set of trainable parameters, which are learned during the training process. Formally, a model $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$ can be expressed as:

$$\hat{y} = f_{\theta}(x) \tag{1}$$

where x is the input data, $\hat{y}$ is the model prediction, and θ represents the trainable param-

eters⁶. The goal of training is to adjust θ to minimize the difference between $\hat{y}$ and the true output y , as measured by a loss function (which will be discussed further in this subsection).

For example, in a linear regression model, $f_{\theta}(x) = w^T \cdot x + b$, where w and b are the trainable parameters.

Data Split

A key focus is on how the algorithm performs on data it has not encountered before, as this determines its generalization ability and real-world applicability. To assess this, we evaluate the performance measure using a test set, a separate dataset that is distinct from the data used for training the model.

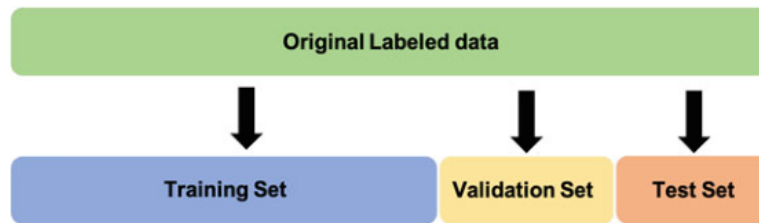


Figure 3: “Available data split into training, validation, and test data sets”. Source: [Ye, 2022].

The available dataset is usually divided into three subsets: training, validation, and test datasets, as Figure 3 shows. The training data is further divided into smaller mini-batches, which are used for stochastic gradient computation during model optimization⁷. The training dataset is used to estimate and update the model’s parameters, while the validation dataset helps monitor the training process and detect potential overfitting [Ye, 2022]. A common split ratio for training and test data is 70:30⁸, taking into account that the validation set is initially contained in the training set.

Model Evaluation

To evaluate the performance of a ML algorithm, we need to define a quantitative measure of its effectiveness. This performance measure, denoted as P , is typically specific to the task T being addressed by the system. Performance is often inversely related to error, with lower error indicating better performance.

⁶The model f can be also be represented as $f(x; \theta)$ instead of the notation used in Equation (1).

⁷Mini-batch and stochastic gradient algorithms are discussed later on this chapter.

⁸Which means the 70% of the available data is used for training purposes, while the remaining 30% is used for testing.

- **Training Error:** Refers to the error calculated on the data used to train the model. While a low training error indicates that the model has learned the patterns in the training data, it does not guarantee good performance on unseen data.
- **Prediction Error:** Refers to the discrepancy between the model's predictions $f(x; \theta)$ and the actual target values y on new, unseen data. This error measures the model's generalization ability and is evaluated using the test set.

Overfitting and Underfitting

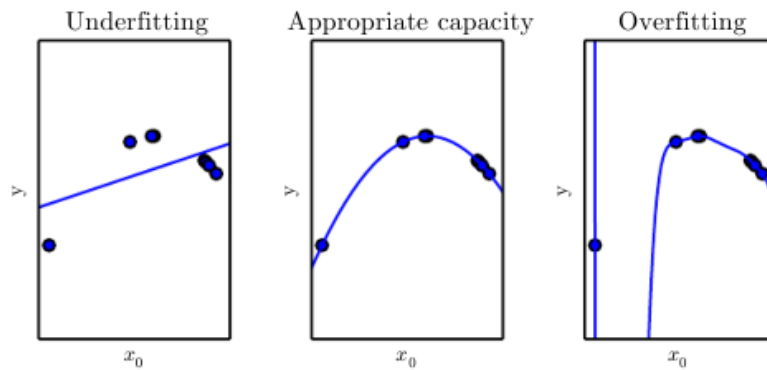


Figure 4: Three plots for the same data set showing underfitting (*Left*), appropriate fit (*Center*), and overfitting (*Right*). x_0 represents values taken randomly while the y is the evaluation of a quadratic function using those points. Source: [Goodfellow et al., 2016].

Underfitting occurs when the model fails to achieve a sufficiently low error on the training set, indicating that it has not learned the task effectively. Conversely, overfitting happens when the model performs well on the training data but exhibits a significant gap in performance when evaluated on the test data, indicating poor generalization. Figure 4 shows an example about how, with a given set of data points, a model can underfit by approximating using a linear model, fit just right using a quadratic function, and overfit using a degree 9 polynomial model.

Loss Functions

Loss functions are essential for guiding the training process by providing a measure of error. They vary based on the task and the nature of the output. Some examples of loss functions are presented in the following list:

- **Loss Functions for Regression:**

- Mean Squared Error (MSE): Penalizes larger errors more heavily by squaring the difference between predicted and actual values.

$$\text{MSE} = \frac{1}{n} \cdot \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

- Mean Absolute Error (MAE): Focuses on the absolute differences between predictions and actual values.

$$\text{MAE} = \frac{1}{n} \cdot \sum_{i=1}^n |y_i - \hat{y}_i|$$

- Loss Functions for Classification:

- Binary Cross-Entropy (Log Loss): Used for binary classification, it measures the difference between the predicted probability and the true label. For example, identify if a picture has a dog or a cat.

$$\text{Binary Cross-Entropy} = -\frac{1}{n} \cdot \sum_{i=1}^n (y_i \cdot \log(\hat{y}_i) + (1 - y_i) \cdot \log(1 - \hat{y}_i))$$

- Categorical Cross-Entropy: Extends binary cross-entropy to multi-class classification problems. For example, identify the race of a dog.

- Custom Loss Functions:

- In advanced models, loss functions can be tailored to incorporate domain-specific requirements, such as penalizing certain types of errors more heavily or addressing class imbalances.

3.2.4 Deep Learning Concepts

DL, a subset of ML, is inspired by the structure and function of the human brain. It involves training Artificial Neural Network (ANN) to learn patterns from data. Key concepts in DL include neurons, layers, and the process of training and learning.

Neurons and Activation Functions

In DL, neurons and activation functions are fundamental components that enable NN to model complex, non-linear relationships in data.

A neuron is the basic computational unit of an ANN. It receives one or more inputs, processes them, and produces an output as can be seen in Figure 5. The operation performed by a neuron i can be expressed mathematically as:

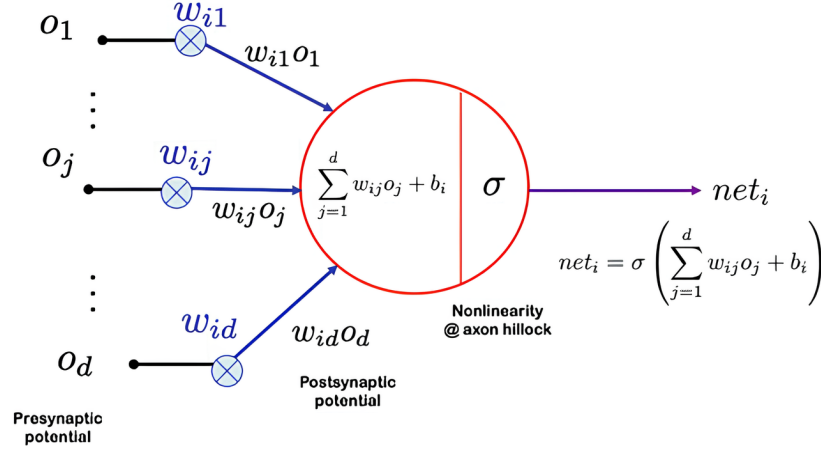


Figure 5: A diagram for the mathematical model of a single neuron. Source: [Ye, 2022].

$$net_i = \sigma \left(\sum_{j=1}^d w_{ij} \cdot o_j + b_i \right) = \sigma(z) \quad (2)$$

Where o_j are the input features, w_{ij} are the weights assigned to each input, b_i is the bias term that allow the model to shift the activation function, z is the weighted sum of inputs, and $\sigma(\cdot)$ is the activation function that produce the neuron output. The weights and bias are trainable parameters that are adjusted during the training process to minimize the error between predicted and true outputs.

Different activation functions are used depending on the type of problem and network architecture, some of them are:

- Linear Activation Function:
 - $\sigma(z) = z$.
 - Used in output layers for regression tasks.
 - Lacks non-linearity, limiting its use in hidden layers.
- Sigmoid Function:
 - $\sigma(z) = \frac{1}{1+e^{-z}}$.
 - Maps the input to a range between 0 and 1.
 - Suitable for binary classification but prone to vanishing gradient problems in deep networks.
- Tanh Function:

- $\sigma(z) = \frac{2}{1+e^{-2z}} - 1$.
- Maps the input to a range between -1 and 1.
- Provides stronger gradients than sigmoid, making it more effective in certain cases.
- Rectifier Linear Unit (ReLU):
 - $\sigma(z) = \max(0, z)$.
 - Outputs zero for negative inputs and a linear response for positive inputs.
 - Efficient and widely used in deep networks, but susceptible to the “dying ReLU” problem where neurons can become inactive [Lu et al., 2019].
- Leaky ReLU:
 - $\sigma(z) = \begin{cases} z & \text{if } z > 0 \\ \alpha \cdot z & \text{otherwise (where } \alpha \text{ is a small constant)} \end{cases}$.
 - Addresses the dying ReLU issue by allowing a small gradient for negative inputs.
- Softmax Function:
 - $\sigma(z_i) = \frac{e^{z_i}}{\sum_j e^{z_j}}$.
 - Converts raw scores into probabilities, often used in the output layer for multi-class classification.

(See the Appendix for the plots of the activation functions).

Architecture and Layers

The term architecture refers to the overall design of an ANN, including the number of units (neurons) that contains and the way these units are interconnected. NN are typically organized into layers, which are groups of neurons that work together. In most NN architectures, these layers are arranged sequentially in a chain-like structure, where each layer’s output serves as the input to the next layer as can be seen in Figure 6.

0

Extending Equation (2) to incorporate the concept of layer, the single neuron model is now expressed as

$$o_i^{(l)} = \sigma \left(\sum_{j=1}^{d^{(l)}} w_{ij}^{(l)} \cdot o_j^{(l-1)} + b_i^{(l)} \right), \quad (3)$$

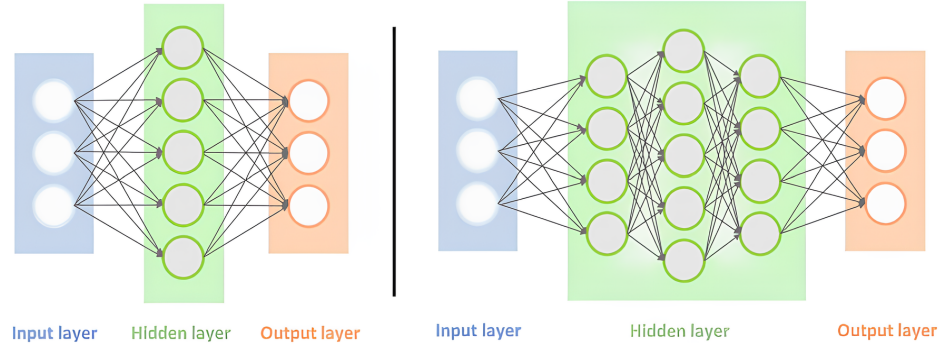


Figure 6: Examples of two Neural Network, one with a single hidden layer (*left*), and other with three hidden layers (*right*). Source: [Ye, 2022].

for $i = 1, \dots, d^{(l)}$, where $d^{(l)}$ corresponds to the number of neurons of the layer l , $o_j^{(l-1)}$ denote the output of the neuron j from the layer $(l - 1)$, and $w_{ij}^{(l)}$ are the weights assigned to each input at the layer l .

The output layer's activation function should align with the learning task. For regression problems, where outputs are continuous, a linear activation function is commonly used, as restricting outputs to a range like $[0, 1]$ may not be suitable ⁹.

3.2.5 Learning with Gradients

Learning in NN is driven by the goal of minimizing a loss function. Gradient-based optimization techniques are central to this process, enabling NN to adjust their parameters and improve performance.

Learning as an Optimization Problem

In [Ye, 2022], Ye define the learning problem as

“ For given training data $\mathcal{D} = \{(\mathbf{x}_n, \mathbf{y}_n)\}_{n=1}^N$ a NN training problem can be then formulated as follows:

$$\hat{\Theta} = \arg \min_{\Theta} c(\Theta), \quad (4)$$

⁹For the interested reader, a comprehensive discussion of the role and significance of the output layer or output units in achieving the tasks of a NN can be found in Subsection 6.2 “Gradient-Based Learning” of Goodfellow et al. book, *Deep Learning* [Goodfellow et al., 2016].

where the cost function is given by

$$c(\Theta) := \sum_{n=1}^N \ell(y_n, f_{\Theta}(x_n)) \quad (5)$$

Where ℓ is the loss function, $f_{\Theta}(x_n)$ is the function that represents the model with the input x_n , which is parameterized by the parameter set Θ .

In simply words, Equation (4) assigns the values for the final set of parameters ($\hat{\Theta}$)¹⁰ to the values that resulted in the minimum value of the cost function ($c(\Theta)$). And Equation (5) defines the cost function as the sum of the deviations between the model with parameters Θ and the real values. So, combining the two equations, the problem is to find the parameters that minimize the deviations between the model and the true values.

Standard Gradient Descent

Gradient Descent is the foundation for learning in NN. It updates the trainable parameters (weights and biases) iteratively to minimize the loss function. Since the gradient points in the direction of the steepest increase in the cost function, the algorithm updates the parameters in the opposite direction to achieve a reduction in the loss:

$$\Theta[t+1] = \Theta[t] - \eta \cdot \nabla_{\Theta} c(\Theta)|_{\Theta=\Theta[t]} \quad (6)$$

where $\eta > 0$ is the learning rate controlling the step size, $\nabla_{\Theta} c(\Theta)$ is the gradient of the cost function with respect to Θ , and $\Theta[t]$ is the t -th update of the parameter Θ .

In each iteration, the algorithm computes the gradient over the entire training dataset, making it computationally expensive and impractical for large datasets.

Standard Stochastic Gradient Descent

To address the inefficiencies of standard gradient descent, Stochastic Gradient Descent (SGD) was introduced. Instead of computing gradients on the entire dataset, SGD approximates them by using a single training example at each iteration. With this, Equation (5) is updated to

$$c(\Theta; x_t, y_t) := \ell(y_t, f_{\Theta}(x_t)) \quad (7)$$

¹⁰The hat in the notation of $\hat{\Theta}$ implies that the parameters were already trained.

where $(\mathbf{x}_t, \mathbf{y}_t)$ is the random example from $\mathcal{D}$ drawn at the t -th iteration. Equation (6) remains the same but with the updated cost function.

While standard SGD improves computational efficiency, it introduces noise, leading to less stable convergence.

Mini-Batch Stochastic Gradient Descent

Mini-batch SGD addresses the noise problem by computing gradients on small, randomly chosen subsets (mini-batches) of B data cases, while retaining the computational efficiency compared to standard gradient descent¹¹. This way, Equation (5) is updated again to

$$c(\Theta; I[t]) := \frac{1}{B} \sum_{i \in I[t]} \ell(\mathbf{y}_i, \mathbf{f}_{\Theta}(\mathbf{x}_i)) , \quad (8)$$

where $I[t]$ is a random subset of the index set $1, \dots, N$ with B elements at the t -th update. A single pass over all the data is called an epoch, and each epoch consists of $\frac{N}{B}$ iterations or updates.

The mini-batch size B is a hyperparameter that significantly impacts learning dynamics. Note that if $B = N$, the standard gradient descent is recovered, and with $B = 1$ the standard SGD is obtained.

ADAM Optimizer

The ADAM algorithm [Kingma, 2014], named after “ADaptive Moment”, is a modern approach built upon the SGD that incorporates additional techniques such as momentum¹² and adaptive learning rates to improve convergence and stability.

3.2.6 Convolutional Neural Networks Concepts

In [Calin, 2020], Calin introduces Convolutional Neural Network (CNN) as “networks with shared weights and sparse interactions, that is, most weights are equal to zero. Given their fewer number of parameters, convolution networks are more efficient to train than any similarly sized fully-connected layer networks, with only minor negative consequences on their performance”.

¹¹In modern hardware (e.g., GPUs) it even facilitates parallelization

¹²Momentum in this context refers to “preserve” the past movement, which translates as using the previous gradients in the algorithm.

The CNNs have the ability to automatically and efficiently learn hierarchical feature representations from data. Originally developed for 2D applications, such as image recognition and computer vision, CNNs are exceptional in detecting spatial patterns like edges, textures, and shapes by leveraging 2D convolutions. In contrast, 1D CNNs are designed for sequential or temporal data, such as time-series signals, audio, or text. While 2D CNNs operate on two-dimensional grids (e.g., image pixels), 1D CNNs focus on extracting patterns along a single dimension, capturing local dependencies and trends in sequences. Despite their structural differences, both 1D and 2D CNNs share the same principles, including convolution, pooling, and weight sharing ¹³.

Kernel and Convolution Operation

In a 1D CNN, the kernel¹⁴ (or filter) is a small, trainable array of weights that slides over the input data to extract features. For example, a kernel of size k is represented as:

$$\mathbf{k} = [w_1, w_2, \dots, w_k],$$

where w_i are the weights. The kernel learns to detect patterns such as edges or trends during training.

The convolution operation is the central piece in a CNN. It involves sliding the kernel across the input and computing the dot product between the kernel and overlapping segments of the input. For an input $\mathbf{x} = [x_1, x_2, \dots, x_n]$ and a kernel $\mathbf{k} = [w_1, w_2, \dots, w_k]$, the convolution is defined as:

$$y_i = \sum_{j=1}^k w_j \cdot x_{i+j-1},$$

where y_i is an element of the output. The convolution is also typically represented as

$$\mathbf{y} = \mathbf{x} * \mathbf{w}.$$

This operation produces an output feature map, which highlights regions where the kernel detects certain patterns.

Stride and Padding

When applying the convolution operation, the stride determines the step size of the kernel as it slides over the input. A larger stride reduces the output size by skipping elements as can

¹³The model designed in this work uses 1D CNNs, so the following definitions will focus on them instead of the 2D CNNs.

¹⁴Do not confuse with a kernel method or function.

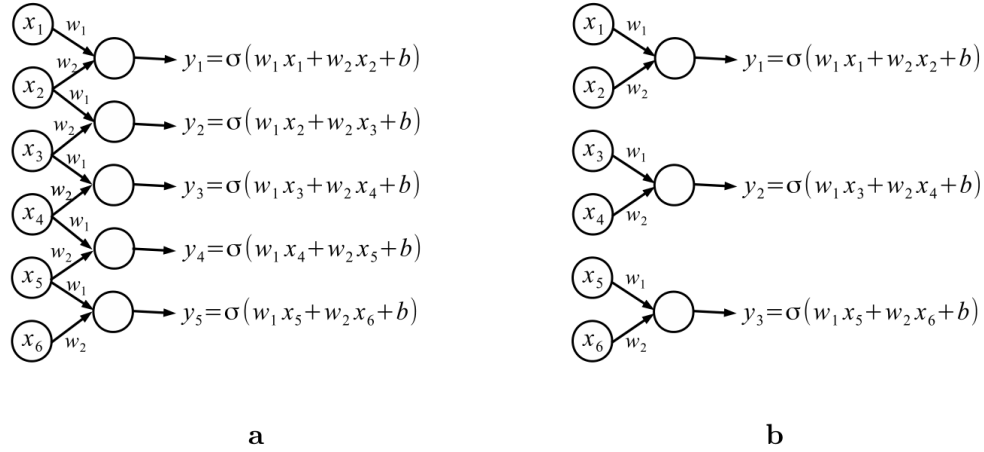


Figure 7: “Two convolution layers: a stride equal to 1; b stride equal to 2. All missing arrows are assigned a zero weight”. Source: [Calin, 2020].

be seen in Figure 7 b. Smaller strides yield more detailed feature maps, while larger strides reduce computational complexity.

Padding refers to adding extra elements (usually zeros) to the edges of the input data before performing the convolution. It controls the size of the output feature map. Padding is essential for preserving spatial dimensions or capturing edge features effectively.

Pooling

In [Calin, 2020], Calin introduces pooling as “a ML technique that provides a summary of the input, selecting some essential local features such as maxima, minima, averages, etc. It also acts as an information contractor; in the discrete case it decreases the dimension of the input by a certain factor. Hence its usefulness in classification problems. The idea of pooling is to consider a partition of the domain of a function and replace the function on each partition element by the ‘most representative’ value of the function on that set”.

Figure 8 shows a signal defined in the interval $[a, b]$ where for each partition the min and the max pooling was evaluated. Note how the use of pooling resulted in a discretized version of the original signal.

Batch Normalization

Batch normalization was introduced to address the issue of internal covariate shift, improving the speed, performance, and stability of ANN [Ioffe, 2015]. During training, changes in the distribution of features in earlier layers can cause shifts in the inputs of subsequent layers.

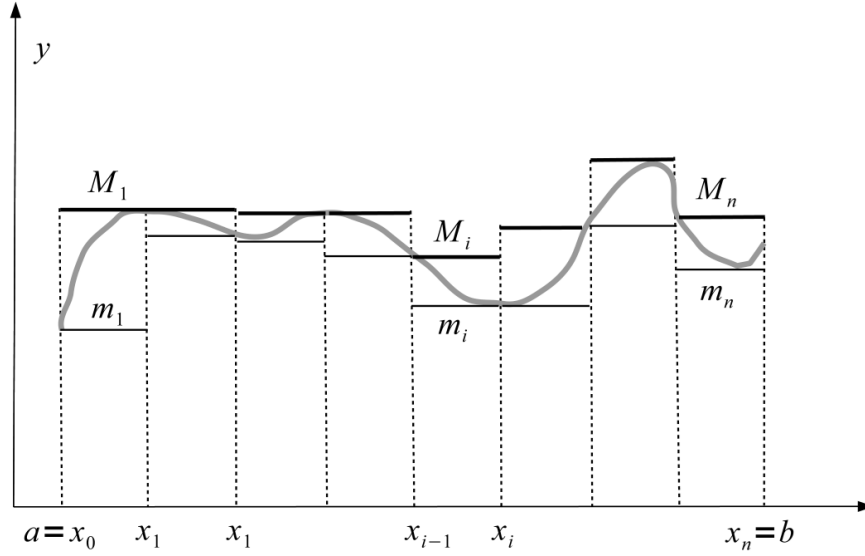


Figure 8: Example of pooling over a signal. M_i represents max pooling, while m_i min pooling. Source: [Calin, 2020].

This requires constant adaptation by each layer to these new distributions, which becomes more pronounced in deep networks as small changes in shallow layers amplify through the network. Batch normalization mitigates this problem by re-centering and scaling the activations, reducing these shifts and enhancing training efficiency.

3.2.7 Anomaly Detection

The task of AD involves identifying unusual or outlier samples within a dataset. It is often approached as an unsupervised learning problem, where the anomalous samples are unknown beforehand, and the majority of the training data is assumed to represent “normal” (non-anomalous) instances¹⁵. The goal is to build a model that effectively characterizes these normal patterns, with deviations from the learned model flagged as anomalies. This approach is also referred to as one-class classification.

3.2.8 Deep Support Vector Data Description

Deep-SVDD [Ruff et al., 2018] is an AD method inspired by kernel-based one-class classification and minimum volume estimation. Unlike other approaches, it is specifically trained using an objective function designed for AD tasks.

Deep-SVDD trains a NN by minimizing the volume of a hypersphere that encapsulates the

¹⁵Here, “normal” refers to typical patterns in the data and is not related to the Gaussian distribution.

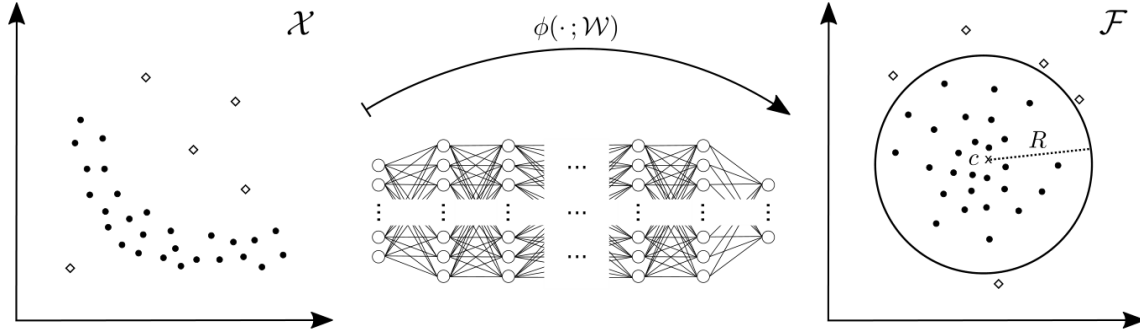


Figure 9: “ Deep Support Vector Data Description learns Neural Network transformation $\phi(\cdot; \mathcal{W})$ with weights $\mathcal{W}$ from input space $\mathcal{X} \subseteq \mathbb{R}^d$ to output space $\mathcal{F} \subseteq \mathbb{R}^p$ that attempts to map most of the data network representations into a hypersphere characterized by center c and radius R of minimum volume. Mappings of normal examples fall within, whereas mappings of anomalies fall outside the hypersphere”. Source: [Ruff et al., 2018].

network’s representations of the data (see Figure 9). This process drives the network to capture the shared factors of variation, as it must map the data points tightly around the center of the sphere.

Deep-SVDD Loss and Cost Functions

As described in Subsection 3.2.3, certain models employ custom loss functions, and Deep-SVDD is one such example. Its loss function, ℓ , is defined as the distances $d(\cdot, \cdot)$ between the center of the hypersphere, $\mathbf{c}$, and a data point $\mathbf{x}_n$ after being mapped to the output space by the NN $\phi(\cdot)$. For the cost function c , in addition to the mean of the loss over the data points $\mathcal{D}$, a regularization term, $R(\mathcal{W})$, is included to evaluate the NN weights $\mathcal{W}$, enhancing the model’s generalization capabilities. The expression of the loss function is then

$$\ell(\mathbf{x}; \mathcal{W}) = d(\mathbf{c}, \phi(\mathbf{x}, \mathcal{W})) , \quad (9)$$

and the expression for the cost function¹⁶ is

$$c(\mathcal{W}) = \frac{1}{N} \cdot \sum_{n=1}^N \ell(\mathbf{x}_n; \mathcal{W}) + R(\mathcal{W}) . \quad (10)$$

¹⁶As mentioned, the brilliance of this method is its objective function, but overly complex math could hinder the main purpose of this document, so more details of the One-Class Deep-SVDD can be found in [Ruff et al., 2018].

Deep-SVDD Evaluation and Performance Metrics

To evaluate the trained model, the radius R of the hypersphere has to be set in order to allow the classification, since the points inside the sphere corresponds to the trained class, any other point outside of the sphere is classified as an anomaly. Here, the distance to the center of the hypersphere acts as an anomaly score, and the value of the radius as the anomaly threshold.

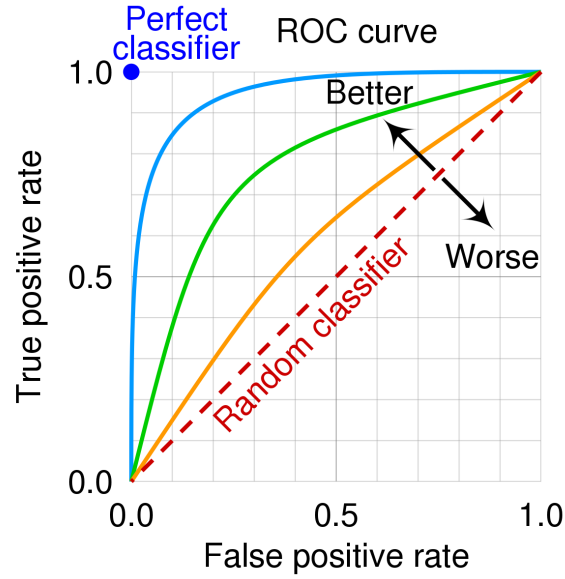


Figure 10: Examples of Receiver Operating Characteristic curves where the classification performance is indicated. The perfect classifier is any curve that pass through the point $(0, 1)$. Source: [Kılıç, 2023].

As a method for a classification problem, the performance metrics that can be used in Deep-SVDD are:

- True Positive Rate (TPR): also known as sensitivity or recall, measures the proportion of true positives correctly identified by the model and is defined as:

$$TPR = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}} .$$

- False Positive Rate (FPR): it quantifies the proportion of false positives out of all actual negatives, reflecting the model's tendency to incorrectly classify negatives as positives. It is given by:

$$FPR = \frac{\text{False Positives}}{\text{False Positives} + \text{True Negatives}} .$$

- Receiver Operating Characteristic (ROC) Curve: A plot of the TPR against the FPR at various threshold levels. It visualizes the trade-off between sensitivity and specificity. See Figure 10 for an explicative example of the use of the curve.

- **Area Under the receiver operating characteristic Curve:** A single scalar value summarizing the ROC curve. An AUC of 1.0 indicates perfect classification, while 0.5 represents random guessing.

In the work by Ruff et al. [Ruff et al., 2018], performance was evaluated using the arithmetic mean and the associated standard deviation of the AUC values obtained after training more than ten independent instances of their models. This approach facilitated the use of statistical measures to ensure reliability and relevance of the results. For a fixed model ϕ , a test data $\mathcal{D}_{\text{test}} = \{(\mathbf{x}_n, \mathbf{y}_n)\}_{n=1}^N$, and a set of parameter sets of trained models $\mathcal{W}^* = \{\mathcal{W}_m\}_{m=1}^M$, the statistical performance s of the model is expressed as:

$$s(\mathcal{D}_{\text{test}}; \phi, \mathcal{W}^*) = \frac{1}{M} \cdot \sum_{m=1}^M \text{AUC}(\mathcal{D}_{\text{test}}; \phi, \mathcal{W}_m). \quad (11)$$

3.3 Physics of Dark Matter

In this section the main source of information is by default the textbook from [De Angelis and Pimenta, 2018], Introduction to Particle and Astroparticle Physics. If other resources are used, they'll be cited accordingly.

In order to understand the nature of the detection of DM, it is crucial to comprehend the underlying processes, particularly the characteristics of the particles and their interactions. When using a detector, not every event detected is a signal of interest, so it is important to distinguish between desired signal events and background events. In DD of DM, various background sources need to be considered. Cosmic rays, for instance, can be mitigated by placing the detector deep underground. However, other backgrounds, such as the natural radioactivity of the detection material and surrounding infrastructure, remain significant, often constituting the most abundant source of interference, although well characterized. Additionally, Neutrino (ν) present a unique challenge, as they interact in a manner similar to DM particles and are impossible to shield against.

The Sun is the main source of neutrino background as it's constantly emitting neutrinos through different processes that are produced inside it, and cannot be shielded because the Sun cannot be turned off, and the neutrinos can travel through the Earth. In the DD of DM, these neutrinos constitute the ν -floor.

3.3.1 Coherent Elastic Neutrino Nucleus Scattering

Scattering and decay are processes induced by interactions, leading from an initial state to a final state. The concept of interaction is thus a generalization of the concept of force ex-

change in classical physics [De Angelis and Pimenta, 2018].

In this context, scattering can be described as a collision or an encounter between particles or group of particles. This interaction is characterized by an incident particle and a target particle, resulting in different by-products. A relevant kind of process is the so-called elastic. “In an elastic process, the particles in the final state and in the initial state are the same—there is simply an exchange of energy-momentum”^{17,18}.

What is a Neutrino?

The Neutrino is a type of fundamental particle that is extremely light, electrically neutral and that interacts very weakly with matter, making them extremely elusive to detect¹⁹. Neutrinos are produced in various astrophysical and terrestrial processes, such as nuclear reactions in the Sun, nuclear reactors, and cosmic ray interactions in Earth’s atmosphere.

The neutrinos are a particle hypothesized by Pauli in 1930 as a solution to the problem of the continuous spectrum of the beta decay (β^-) that deviated from the previous predictions of the existing models. The spectrum was explained by introducing the neutrino as a neutral, weakly interacting particle emitted alongside beta particles, carrying away energy and momentum

$${}^A_Z N \rightarrow {}^A_{Z+1} N' + e^- + \bar{\nu}_e .$$

β^- processes transform an atomic nucleus (${}^A_Z N$) into the next element of the periodic table (${}^A_{Z+1} N'$), while emitting an electron (e^-) and an electron antineutrino ($\bar{\nu}_e$). This is the process that motivated the existence of the neutrino. Today, the neutrino is known for being an essential “ingredient” of electroweak processes.

The reaction described above is called β^- decay because the final state contains an electron. In this case an electron antineutrino is produced. But when the final state of the decay has a positron (e^+), the second particle produced is an electron neutrino (ν_e). This reaction is different given the final state, thus called β^+ decay and is the one that mainly occurs inside the Sun

$${}^A_Z N \rightarrow {}^A_{Z-1} N' + e^+ + \nu_e .$$

¹⁷Cited directly from the page 42 of [De Angelis and Pimenta, 2018].

¹⁸Strictly speaking, for a process to be considered elastic, the 3-momentum and energy should remain unchanged. However, in Coherent Elastic Neutrino-Nucleus Scattering, the final kinematic states do change. Despite this, the process is still referred to as elastic because the nucleus recoils as a whole, without excitation, and for historical reasons.

¹⁹For the phenomenological understanding of this work, neutrino flavor can be ignored since the detectors of interest are not sensitive to those quantum numbers.

Neutrino Solar Production

The fundamentals of solar neutrino physics are comprehensively discussed in the article by Xu et al. (2023) [Xu et al., 2023]. This article provides an in-depth analysis of the sources, detection methods, and interaction mechanisms of solar neutrinos. Some contents of the article are used in the following content.

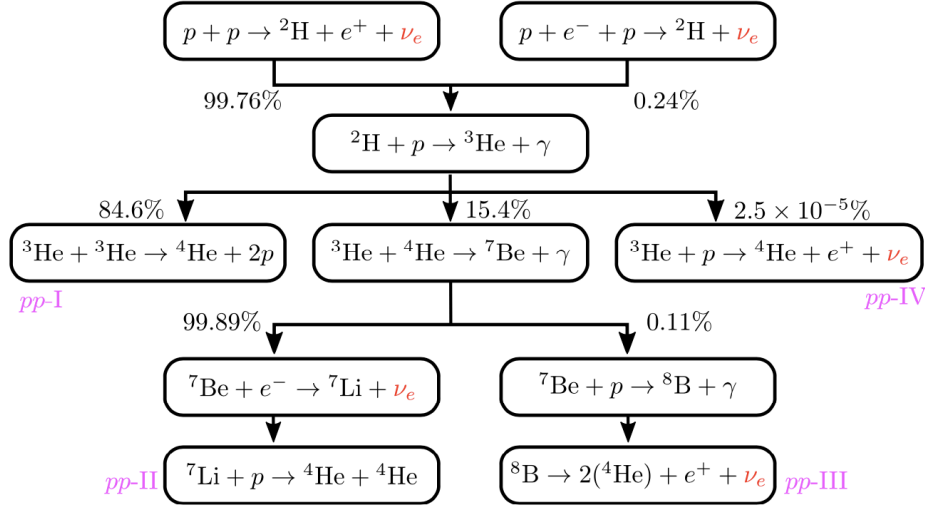


Figure 11: Reactions that constitute the solar proton-proton chain with their corresponding theoretical branching rate. Image source: [Xu et al., 2023].

Two primary sets of nuclear reactions are responsible for the production of neutrinos and energy in the Sun: the proton-proton chain (pp chain) and the Carbon-Nitrogen-Oxygen cycle (CNO cycle). The pp chain generates approximately 99% of the Sun's total energy, while the CNO cycle contributes the remaining $\sim 1\%$. The total flux of neutrinos from the Sun must align with the solar luminosity in photons, assuming all fusion processes are accurately accounted for.

As illustrated in Fig. 11, the pp chain includes five reactions that produce neutrinos with names based on their initial particles: pp , pep , hep , ${}^7\text{Be}$, and ${}^8\text{B}$ neutrinos. The pp chain comprises four sub-chains, labeled pp -I to pp -IV. Despite some intermediate stages involving heavier elements, all sub-chains ultimately produce ${}^4\text{He}$. Hence, the pp chain essentially converts hydrogen into helium. The first three sub-chains (pp -I, pp -II, and pp -III) produce most of the energy and the neutrinos generated in the pp chain. The fourth sub-chain (pp -IV), while contributing a negligible amount (10^{-5}) to the total energy production, produces the most energetic neutrinos (hep neutrinos) with energies up to 18.77 MeV.

In the CNO cycle, carbon and nitrogen act as catalysts, meaning their abundances remain nearly unchanged after a complete cycle of reactions. As depicted in Fig. 12, ${}^{12}\text{C}$ captures a proton and transforms into ${}^{13}\text{N}$, which then decays to ${}^{13}\text{C}$. This cycle of reactions continues as ${}^{13}\text{C} \rightarrow {}^{14}\text{N} \rightarrow {}^{15}\text{O} \rightarrow {}^{15}\text{N}$. The final element, ${}^{15}\text{N}$, predominantly reverts to ${}^{12}\text{C}$.

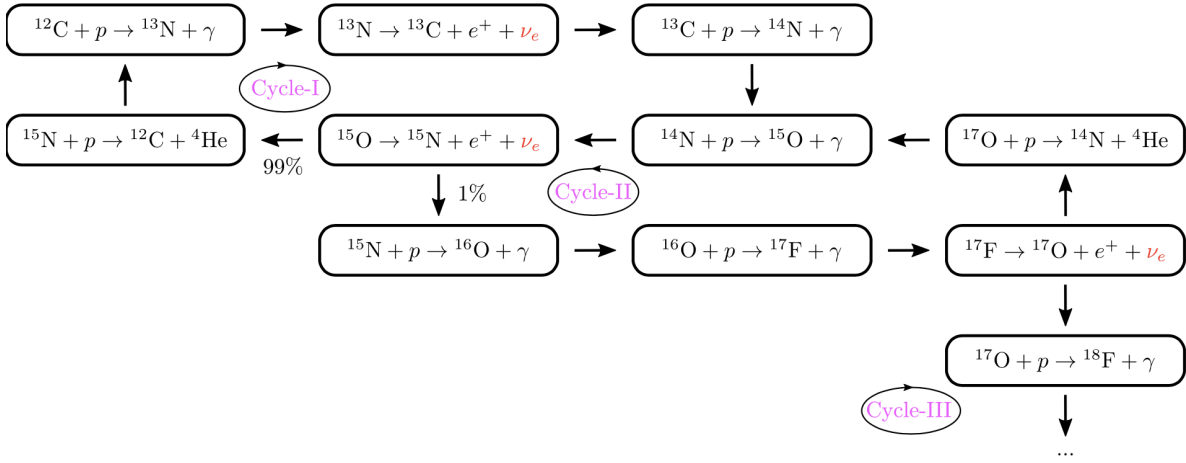


Figure 12: Reactions that constitute the Carbon-Nitrogen-Oxygen cycle. Cycle-III and subsequent cycles contributions are almost negligible for the Sun. Image source: [Xu et al., 2023].

This sequence, referred to as Cycle-I, produces neutrinos through the β^+ decays of ^{13}N and ^{15}O . As with the pp chain, the CNO essentially produces helium while generating energy and emitting neutrinos. In the cycle, there's a small branching ratio where ^{15}N can be converted to ^{16}O , entering a less dominant cycle, Cycle-II, which generates ^{17}F and an additional source of neutrinos. As the branching ratio of Cycle-II is suppressed by approximately two orders of magnitude, the neutrino flux from ^{17}F is significantly lower compared to the neutrino fluxes from ^{13}N and ^{15}O .

Fig. 13 illustrates the neutrino flux generated by each reaction previously described in the pp chain and the CNO cycle. The figure also depicts elements of the CNO cycle that are also subject to electron capture (e.g. $^{17}\text{F} + e^- \rightarrow ^{17}\text{O} + \nu_e$), in this process there is also neutrino production, but the fluxes generated are suppressed by a factor of $\sim 10^{-4}$ compared to the β^+ decay neutrino fluxes.

Relevant Processes in Neutrino Detection

When working with detectors, the relevant processes are the ones that can be detected. Thus, with neutrinos being the main source of background events, it is important to study these processes among which-in the energy regime we are concerned-one can mention: the elastic neutrino-electron scattering, and Charged Current (CC) neutrino-nucleus scattering.

Elastic neutrino-electron scattering, Equation (12), is a fundamental process in which a neutrino interacts with an electron, transferring some of its energy and momentum to the electron. This process is characterized by the conservation of energy and momentum. A fraction of the neutrino four-momentum is transferred to the electron. A measurable signal is the one related with the electron recoil.

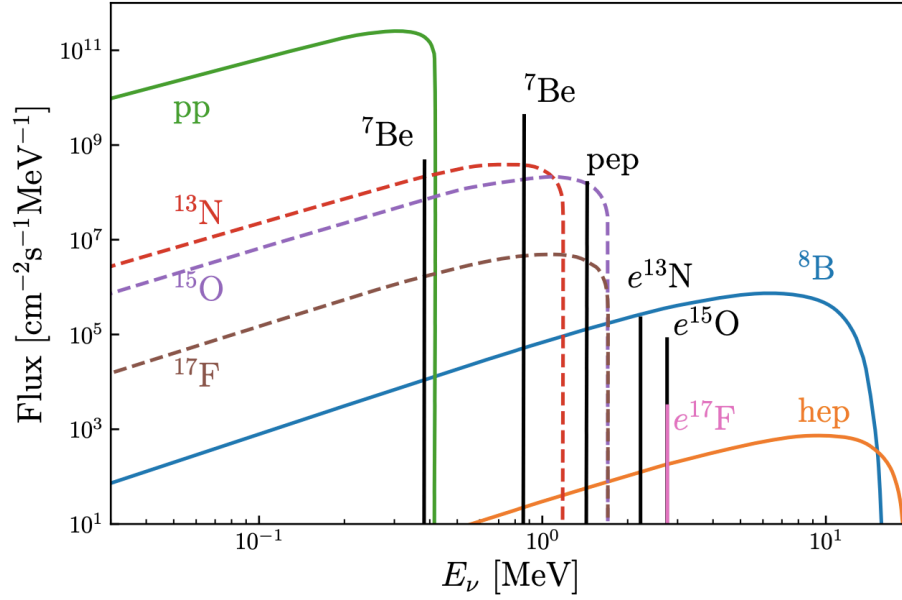


Figure 13: “The solar neutrino spectra predicted by the Standard Solar Model. Monochromatic spectra are given in units of $\text{cm}^{-2}\text{s}^{-1}$. Dashed lines denote neutrinos from the Carbon-Nitrogen-Oxygen cycle”. Image source: [Xu et al., 2023].

$$\nu_e + e^- \rightarrow \nu_e + e^- . \quad (12)$$

When solar neutrinos scatter on electrons in a detector, only the final-state electron is observable. This method has the disadvantage that neutrino energy can only be constructed on a global basis since the energy and direction of the electron are not well measured in an event-by-event scale. For example, water Cherenkov detectors are able to measure direction up to a certain point, while liquid scintillator detectors cannot measure the direction with enough precision.

CC neutrino-nucleus scattering is another important process in neutrino detection. In this process, a neutrino interacts with a nucleus via the weak nuclear force, leading to various outcomes depending on the neutrino energy and the target nucleus. This process typically has a higher cross-section²⁰, especially at higher neutrino energies. In addition, the coupling to nuclei is larger than to electrons. This leads to an increase of the probability of a neutrino interacting with a nucleus inside a detector.

$$\nu_e + {}^A_ZN \rightarrow {}^A_{Z+1}N' + e^- . \quad (13)$$

²⁰In a quantum scattering process, the cross-section is a measure of the probability that a specific interaction or scattering event will occur between particles. It essentially quantifies how “large” a target appears to an incoming particle, analogous to the target area in a classical sense but defined in a quantum mechanical framework.

“At solar neutrino energies, CC scattering of neutrinos with a nucleus usually cannot break the nucleus, simply converting one of the neutrons in the nucleus to a proton,

$$\nu_e + n \rightarrow p + e^- ,$$

which we refer to as Neutrino-capture Beta Decay (ν BD)²¹. In a practical ν BD process, as neutrons are unstable, they have to be bound in a nucleus to produce the decay, giving Equation (13). This process has been used in solar neutrino detection, for example, the Homestake experiment [Davis Jr et al., 1968] used the reaction $\nu + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$, and successfully extracted and counted the final state nuclei using radiochemical methods.

Quantum Coherence and Coherent Elastic neutrino-Nucleus Scattering

In the context of this work, given a system of multiple particles (an atomic nucleus for example) coherence corresponds to a maximum constructive interference of a measurable quantity with quantum nature (the individual nucleon wave functions add up constructively).

Given a neutrino traveling with energy $E_\nu \lesssim 50$ MeV, elastic scattering with a nucleus via Neutral Current (NC) interactions turns out to be coherent, which implies that the cross-section can be substantially enhanced by the large number of neutrons in the nucleus. This process is known as Coherent Elastic Neutrino-Nucleus Scattering (CE ν NS), Eq. (14) [Xu et al., 2023]. In coherent scattering, the neutrino interacts with the entire nucleus coherently, meaning that the phase of the scattering amplitude is the same for all nucleons in the nucleus. As a result, the CE ν NS process has a collective effect on the entire nucleus, similar to a single coherent scattering event.

$$\nu_e + {}^A_Z N \rightarrow \nu_e + {}^A_Z N . \quad (14)$$

The coherence in CE ν NS can be thought as the neutrino interacting with the nucleus as a whole, instead of scattering with all its constituents individually. This is caused by the energy of the incident neutrino which at low enough energies, it can only interact with the nucleus as a whole instead of accessing its components. The process can also be thought as the wave functions of each nucleon interfering constructively, boosting the ‘area’ (cross-section) in which the incident neutrino can interact.

The downside of CE ν NS is the low nucleus recoil energy ($\sim$ keV for 10 MeV neutrinos), thus implying a small signal. making it challenging to detect using this process, despite the fact of the boosted cross-section. For example, energies of this size are completely undetectable at liquid scintillator or water Cherenkov detectors. Although for the resolution of the DM detectors, it could be possible to detect solar neutrinos using this method.

²¹Cited directly from [Xu et al., 2023].

3.3.2 WIMP-Nucleus Interactions

To understand the modelling of the DM, it's necessary to introduce the current Standard Cosmological Model or Λ Cold Dark Matter (Λ CDM). The Λ CDM model, is a version of the Big Bang cosmological model based on general relativity with a simplified set of parameters.

The Universe is primarily composed of dark energy, represented by a nonzero cosmological constant Λ , and cold dark matter²². The Λ CDM model assumes that the Universe is homogeneous and isotropic, and it follows a power law spectrum for primordial fluctuations. This model effectively explains the existence and structure of the Cosmic Microwave Background (CMB), the large-scale distribution of galaxies, nucleon abundances, and the accelerating expansion of the Universe.

Dark Matter

The DM is hypothetical form of matter proposed as the solution to several phenomena in astrophysics. There's missing mass that cannot be seen in the Universe according to the gravitational effects that are appreciated by scientists. This and other observations like the gravitational lensing and the large scale structure formation are well explained by the DM. So, if the hypothesized DM is found, these phenomena will also be successfully explained²³.

DM does not emit, absorb, or reflect light (electromagnetic radiation), making it invisible and detectable only through its gravitational effects. It is thought that "its abundance in the Universe amounts to about an order of magnitude more than the conventional matter we are made of"²⁴, matter that makes up stars, planets, and galaxies. DM interacts very weakly, if at all, with ordinary matter and itself, which is why it is difficult to detect directly. For a good introduction to the physics of DM see the article by Lisanti (2016) [Lisanti, 2016].

Where can the Dark Matter be found?

Despite its weak interactions, DM can be searched given its nature and the knowledge scientists have about the astronomical objects that exists in the Universe. Since DM only causes gravitational effects, there is a clear place where it could be found, near the Super-Massive Black Hole (SMBH) at the center of our Galaxy²⁵. Due to the huge gravitational attraction the

²²In the context of the early Universe, "cold" refers to non-relativistic movement, meaning they moved slowly compared to the speed of light.

²³There are other hypotheses that explain some of these phenomena, but those hypotheses don't explain every one of these as well as the DM does.

²⁴Cited directly from the page 18 of [De Angelis and Pimenta, 2018]

²⁵Sagittarius A* is the SMBH located at the center of the Milky Way.

SMBH exerts, it's expected to find a very large amount of DM caught near it, and accumulating around other black holes.

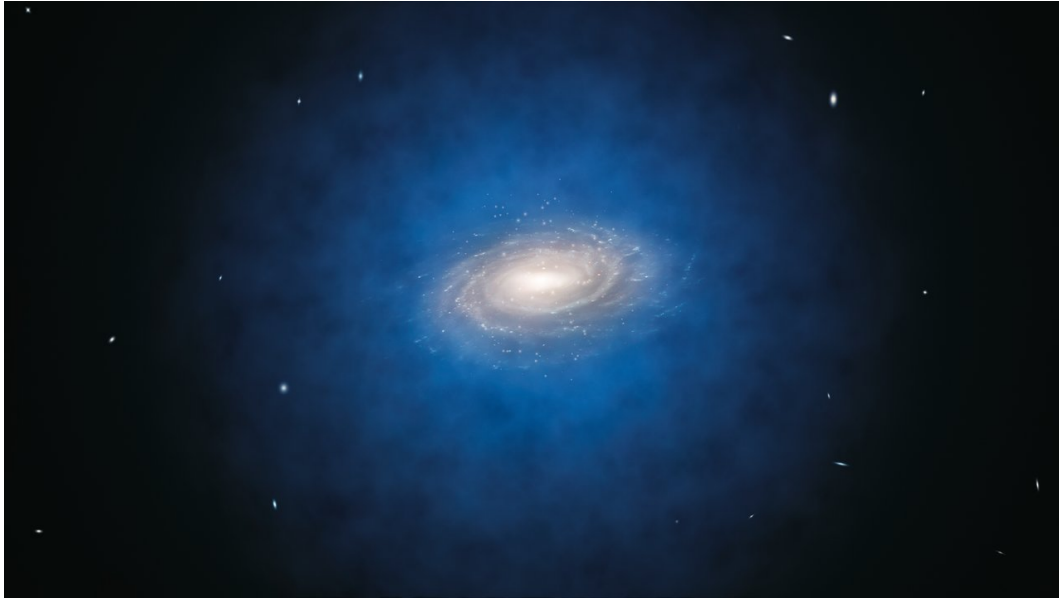


Figure 14: Artist impression of the Milky Way embedded in its Dark Matter halo (in blue). The depiction shows the expected distribution of Dark Matter. Image source: ESO/L. Calçada.

Given the observation of the rotation curves of spiral galaxies²⁶, it is believed that DM form a halo around galaxies, extending well beyond the visible edge as depicted in the Fig. 14. This halo could account for the difference between the observed rotation curves and the predictions based only on visible matter. Since the Earth is immersed inside this halo, DM should be detectable through direct detection. Detectors are sensitive to the relative velocity between the Earth and the DM particles. Around June, the Earth's orbital motion around the Sun aligns with the Sun's motion through the Galaxy, resulting in the highest relative velocity between the Earth and the DM halo. This increased velocity enhances the flux of DM particles potentially hitting the detectors.

How is the Dark Matter searched?

DM is searched through several approaches, each one of them using different aspects of the properties and interactions of the DM. The approaches are categorized into DD, indirect detection, and direct production.

The DD aims to observe DM particles interacting with normal matter directly. It applies highly sensitive detectors placed deep underground, often beneath mountains, to avoid cosmic

²⁶The rotation speed of stars in spiral galaxies does not decrease with distance from the Galactic Center as expected from visible matter alone, suggesting the presence of DM.

radiation. These detectors are used to detect the recoil of a nucleus after being hit by a DM particle²⁷.

The experiments labeled as indirect detection aim to detect the byproducts of DM annihilation within our Galaxy or beyond. Although DM annihilation is significantly reduced after thermal freeze-out²⁸, it can still occur today, making regions with very high DM density ideal for discovery efforts. Depending on the theoretical model, DM can either annihilate directly into photon pairs or into other SM particles that subsequently produce photons through secondary interactions. These gamma rays then travel largely undisturbed, potentially reaching and being detected by a satellite or ground-based telescope on Earth [Lisanti, 2016].

DM particles can be produced at particle colliders, but detecting them is challenging due to their neutral and weakly interacting nature. Nevertheless, their presence can be inferred. The signature of the particles would be missing momentum when reconstructing the dynamics of a collision that produces both DM particles and SM particles. Notice that collisions that produce only DM particles would not be detected [De Angelis and Pimenta, 2018].

What is a WIMP?

“The Λ CDM model assumes that DM is formed by stable massive non-relativistic particles. These particles must have an interaction strength weaker than the electromagnetic one, otherwise they would have been found; the acronym Weakly Interacting Massive Particle (WIMP) is often used to name them”²⁹.

WIMPs being the leading candidate for the DM particle have the following main characteristics:

- **Massive:** WIMPs are believed to have a mass comparable to that of atomic nuclei, typically in the range of $10 \text{ GeV}/c^2$ to several TeV/c^2 .
- **Weakly Interacting:** They interact with other particles primarily through gravity and potentially through a new weak interaction. This weak interaction makes them difficult to detect directly because they do not emit, absorb, or reflect light.
- **Stable:** For WIMPs to account for the DM observed in the Universe, they must be stable on cosmological timescales, meaning they do not decay over billions of years.
- **Non-relativistic:** In the early Universe, WIMPs were cold dark matter, moving slower than the speed of light. This slow movement allowed them to clump together and form the structures we observe today, like galaxies and galaxy clusters.

²⁷The DD will be further explained later in this subsection.

²⁸“Freeze-out” is a term used in cosmology and particle physics to describe a critical phase in the early Universe when certain particles, such as DM particles, stop interacting significantly with other particles and their density or abundance becomes fixed or “frozen”.

²⁹Cited directly from the page 522 of [De Angelis and Pimenta, 2018].

Interaction between Nuclei and WIMPs

DD of DM relies on the interaction between atomic nuclei and DM particles (WIMPs). This interaction is expected to occur through an elastic scattering process between the WIMP and the nucleus and its nucleons.

The WIMP-nucleon scattering cross-section consists of a Spin-Dependent (SD) component and a Spin-Independent (SI) component. When the scattering is coherent, the SI cross-section scales quadratically with the mass number (A^2), resulting in a significant enhancement for heavier elements. However, a nucleus can only recoil coherently if $A \ll 50$. In contrast, SD scattering depends on the total nuclear angular momentum. For SD interactions, the cross-section is smaller by a factor proportional to $A - A^2$ compared to coherent scattering.

3.3.3 Dark Matter Detectors

“Experimental detection is based on the nuclear recoil that would be caused by WIMP elastic scattering” [De Angelis and Pimenta, 2018].

Direct Detection

DD refers to the experimental approach of observing DM particles (WIMPs), by detecting their interactions with normal matter. This method aims to measure the rare and weak interactions between DM particles and atomic nuclei in a detector.

The DD is grounded in physical principles and assumptions of the properties and behavior of DM, the main ones that justifies its search through detectors in Earth being the velocity of the DM and the flux of particles on Earth. WIMPs have velocities around Earth that are one order of magnitude smaller than the Galactic escape velocity, meaning that they can't escape the gravitational pull of the Milky Way. Despite the small probability of WIMPs scattering off nuclei due to their weakly interacting nature, the large flux of WIMPs passing through the solar system ensures that a significant number of interactions can still occur, making it feasible for direct detection experiments to observe and study these events.

The Earth's orbit around the Sun and the Sun's motion through the Galaxy contribute to seasonal variations in the relative velocity between DM particles and the Earth, leading to annual modulation signals in DD experiments. In other words, the Earth's velocity around the Sun adds to or subtracts from the velocity of the solar system with respect to the Galactic Center (GC). As seen in Figure 15, the two velocities implied sum up in June.

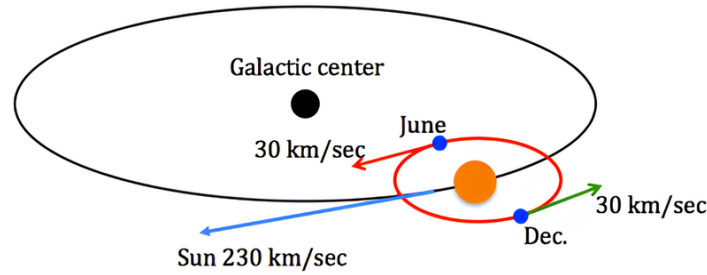


Figure 15: Diagram of the annual modulation depicting the Solar Velocity with respect the Galactic Center, and the Velocity of the Earth during June and December with respect to the Sun. Image source: [COSINE-100, 2016].

Characteristics of a Dark Matter Detector

A DM detector is a specialized scientific instrument designed to directly detect interactions with DM particles, this being very hard to detect. To increase the probability of detection, the detectors should have a low energy threshold (for detecting low energy particles), a low-background noise (to increase the region of possible detections), and a large mass (to increase the probability of interaction).

The primary challenge in the DD of DM is distinguishing genuine nuclear recoils from the overwhelming background of other signals. To address this, detectors must be well-shielded from environmental interference and made from materials with very low radioactive activity. This is mostly achieved by constructing the detectors in underground laboratories, where they are shielded from cosmic rays and natural radioactivity. The detectors also keep the detection material (the one that is going to scatter with WIMPs) at very low temperatures to reduce thermal noise and increase sensitivity to low-energy interactions.

There are three main types of detectors:

- Semiconductor detectors, typically made from materials like germanium or silicon, are highly sensitive to small energy deposits and provide excellent energy resolution through ionization signals, often using a technique to increase the background rejection by keeping the detector in cold³⁰.
- Scintillating crystal detectors, such as those using sodium iodide or cesium iodide, emit light (scintillation) when particles interact with them. These detectors are low cost in comparison, but also have low resolution, thus they are not used in event-by-event analysis.
- Noble liquid detectors, utilizing substances like liquid xenon or argon, combine the advantages of both ionization and scintillation detection, offering high sensitivity, large

³⁰Bolometric technique.

target masses, and the ability to discriminate between different types of interactions. These detectors take advantage on the material low background from itself by being noble elements.

The best current technique is the use of noble liquid, combined with a double phase technique (Figure 16). A vessel is partially filled with noble liquid, while the remaining volume contains the same element in its gaseous state. Electric fields of approximately 1 kV/cm and 10 kV/cm are applied across the liquid and gas volumes, respectively. When an interaction occurs in the liquid, it generates excitation and ionization processes. Photomultiplier tubes are located in both the gas and liquid volumes. This double-phase configuration allows for the reconstruction of the interaction topology (with the gas enabling Time Projection Chamber (TPC) reconstruction), which aids in background removal.

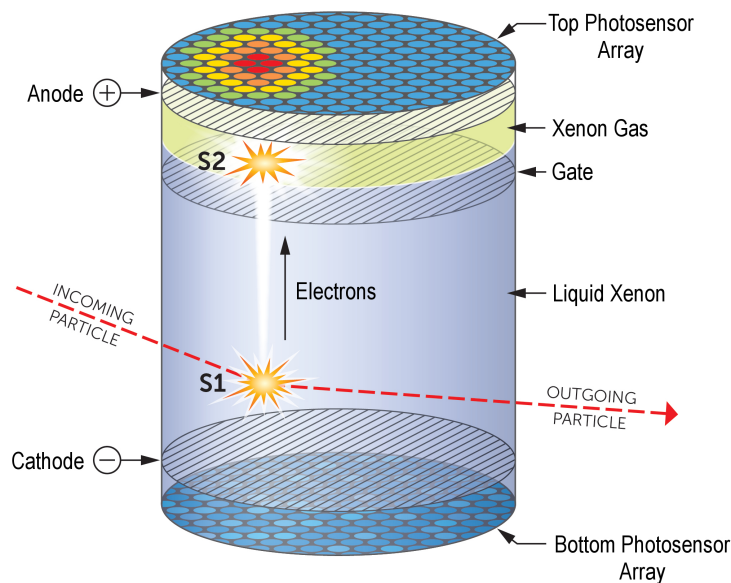


Figure 16: Principle of a dual-phase liquid xenon Time Projection Chamber: When a particle interacts within the active liquid xenon volume, it generates an initial scintillation light signal (S1) followed by a delayed signal (S2) resulting from electroluminescence (proportional scintillation) in the gaseous xenon layer. By analyzing the localization of the S2 signal and the time difference between S1 and S2, the original interaction vertex can be accurately determined. Image source: [Aalbers et al., 2022].

Over the years, direct detection experiments have progressively increased the amount of detection material used in their detectors to enhance the probability of capturing rare DM interactions. A prime example of this trend can be seen in xenon-based experiments. Early xenon detectors, such as XENON10, began with a modest amount of liquid xenon to demonstrate the feasibility of detecting DM particles through scintillation and ionization signals. As techniques and technologies advanced, larger experiments like XENON100 and LUX³¹ followed, utilizing significantly more liquid xenon to improve sensitivity and reduce statistical

³¹<https://sanfordlab.org/large-underground-xenon-lux-dark-matter-detector>.

uncertainties. This trend continued with the development of XENON1T, which employed over a ton of liquid xenon, marking a substantial leap in detection capability. The latest iteration, XENONnT³², along with other large-scale projects like LZ³³, now deploys several tons of liquid xenon, enabling unprecedented detection sensitivity. By scaling up the detector mass, these experiments enhance their ability to detect the rare events associated with DM interactions, thereby pushing the boundaries of current knowledge [XENONnT, 2020].

3.3.4 New Neutrino Interactions

With the construction of Large Particle Colliders and detectors put around them, subnuclear particles (quarks) were discovered and a SM was built, piece by piece, until its final prediction was confirmed with the recent discovery of the Higgs boson³⁴.

As previously discussed, the primary source of background noise in the detection of DM particles comes from neutrinos (the ν -floor). Therefore, a deep understanding of neutrinos and their interactions is essential. The currently accepted theory that describes neutrinos is contained in the SM. However, new theories about additional interactions involving neutrinos are being developed. This is why it's important to explore the possibility of detecting these BSM physics within DM detectors, which are highly sensitive.

Interactions inside Standard Model of Particle Physics

At the energy scales present in the Universe today³⁵, particles interact through four fundamental interactions³⁶, each one having a “charge”³⁷ that allow particles to interact through them:

- **Electromagnetic Interaction:** acts between any pair of electrically charged particles, being dominant at atomic scales or small molecular scales ($\sim 1\text{nm}$), and having a $1/r$ dependence with the distance. This interaction has the well known electric charge, being either positive or negative.
- **Strong Interaction:** also called Color Force or Nuclear Strong Force, it acts between subnuclear particles, being dominant at nuclei scales ($\sim 1\text{fm}$). This interaction has three types of charge, the red, green and blue.

³²<https://xenonexperiment.org>.

³³<https://sanfordlab.org/news/lux-end-era>.

³⁴Adapted from the page 3 of [De Angelis and Pimenta, 2018].

³⁵Higher energies represent older epochs, and current epochs have lower energy compared to the ones at the beginning of the Universe.

³⁶To avoid repetitiveness the words force and interaction are used interchangeably.

³⁷The term charge is used to refer to quantum numbers that each interaction is characterized with.

- **Weak Interaction:** also called Nuclear Weak Force, affects all matter particles, having effect only at subatomic distances ($\sim 1\text{fm}$). The charge of this interaction is called “weak isospin”.
- **Gravitational Interaction:** acts between whatever pair of bodies, being dominant at macroscopical, astrophysical and large scales, negligible at small scales, and having a $1/r^2$ dependence with the distance. This interaction has a charge proportional to the mass (energy).

The strong and weak nuclear forces, which operate over extremely short distances, only have significant effects at subnuclear scales. In contrast, the electromagnetic and gravitational forces, both following a $1/r$ law, can act over distances that we can perceive. While the electromagnetic force is much stronger than gravity at smaller scales, the Universe is electrically neutral on large scales, causing gravitational interactions to dominate the evolution of the Universe.

Particles and Force Carriers

In modern Quantum Mechanics (QM), particles are described as fields—complex amplitudes associated with points in spacetime to which local probabilities can be assigned. In the framework of Quantum Field Theory (QFT), interactions between elementary particles are mediated by force-carrying fields, which can also be interpreted as particles themselves.

The SM classifies particles into two main categories (see Figure 17): matter particles, or fermions, and force-carrying particles, or bosons. This classification is based on a property called spin. Fermions have half-integer spin (e.g., $1/2$) while bosons have integer spin (e.g., 1).

Fermions are further divided into two families: leptons and quarks. Each family consists of three generations, with similar properties but different masses. The first generation of leptons includes the electron and the electron neutrino, while the heavier generations consist of their counterparts. Similarly, quarks are composed of the first-generation up and down quarks, along with their heavier counterparts in the subsequent generations. The up and down quarks are subnuclear particles, the fundamental building blocks of protons and neutrons, which form atomic nuclei.

Bosons are the so-called force carrier particles, being the quanta of each interaction field:

- **Electromagnetic Interaction:** Mediated by the photon field, which is the quantum of this interaction. The photon is massless, electrically neutral, and has a spin of 1.
- **Strong Interaction:** Mediated by gluons as its quanta, with eight different types corresponding to different fields. Gluons are massless, neutral, and have a spin of 1.

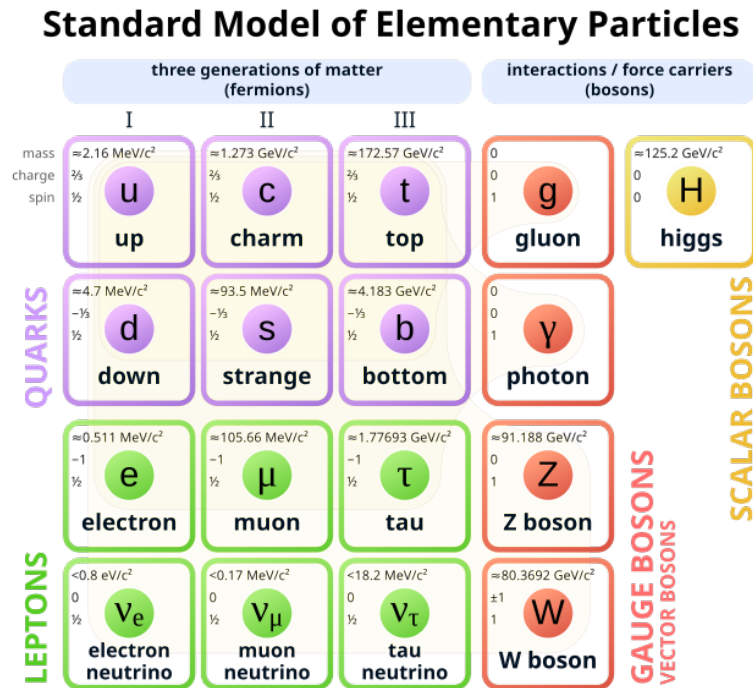


Figure 17: Observed elementary particles. The 12 fundamental fermions are presented in the first three columns, the 4 gauge bosons are in the fourth columns, and the Higgs boson stands in the fifth columns. Image source: [MissMJ, 2024].

- **Weak Interaction:** Mediated by the W^+ , W^- , and Z bosons, each associated with its own field. These bosons have mass and a spin of 1. The Z boson is neutral, while the $W^\pm$ bosons carry an electric charge.
- **Gravitational Interaction:** Hypothetically mediated by the graviton, a particle yet to be discovered. The “graviton” is expected to be massless, neutral, and have a spin of 2.

The gluon, photon, Z and W bosons are all vector bosons because they have a spin equal to one³⁸. Meanwhile, the scalar bosons have spin equal to zero like the Higgs.

Some Problems in the Standard Model

The SM, as initially formulated, was a massive achievement in the effort of describing the fundamental forces, providing a useful framework for physicists. However, over time, experimental discoveries shown that the SM needed modifications. Particularly, in the context of neutrinos, one of the key challenges was the solar neutrino problem.

³⁸“The vector boson has a spin of unity” [Brianti and Gabathuler, 1983].

Between the 1960s and 1970s, experiments and detectors that measured the neutrinos from the Sun, observed fewer neutrinos than predicted by the Standard Solar Model (SSM). This discrepancy is known as the solar neutrino problem, and posed a challenge to the SM, which treated neutrinos as massless particles.

The resolution of the solar neutrino problem came with the discovery of neutrino flavor oscillations, the phenomenon where neutrinos change from one “flavor” (electron, muon, or tau) to another as they propagate³⁹. This process is only possible if neutrinos have mass, contradicting the original assumption of the SM. Consequently, the SM had to be extended to include non-zero neutrino masses. This modification did not fundamentally alter the structure of the SM but opened the door to new physics.

While the addition of neutrino mass was an important step, it also raised new questions that the SM cannot fully address. For instance, the mechanism behind the neutrino mass generation is still unknown, as is the nature of neutrinos⁴⁰. These open questions suggest that there may be physics beyond the SM.

Beyond Standard Model and New Physics

While the SM provides a robust framework for understanding neutrino interactions via the weak nuclear force, the discovery of neutrino mass and flavor oscillations suggests that the SM is incomplete. This opens the possibility for BSM physics, where neutrinos could interact in ways not predicted by the SM.

Recalling that neutrinos are the primary source of background noise in DM detectors, the presence of new neutrino interactions could result in unexpected measurements and complicate the detection of DM.

New physics, in the form of new interactions, can affect experimental measurements by altering the cross-sections of key processes inside the detectors. For instance, some models propose that neutrinos might undergo NSI with matter, leading to a rescaling of the SM cross-section. Interactions mediated by light scalar or vector bosons introduce additional momentum transfer, potentially producing distinctive spectral features. Vector and scalar Neutrino Generalized Interactions (NGI), depending on their nature, have different implications in the neutrino behavior, leaving distinct signatures that can be observed in experiments [Aristizabal Sierra et al., 2019].

³⁹“The transmutation of neutrinos from one species to another implies in a quantum mechanical world an oscillation phenomenon”. (Cited directly from the page 549 of [De Angelis and Pimenta, 2018]).

⁴⁰The question about the nature of the neutrinos is whether they are Majorana or Dirac particles. A Dirac particle is distinct from its antiparticle (i.e. electron and positron). In contrast, a Majorana particle is its own antiparticle, meaning the neutrino and antineutrino would be the same.

Chapter 4

Proposed Solution

This chapter provides an in-depth review of all the work completed in this project, covering every step from the initial data exploration to the evaluation of the final models. An overview of the proposed solution and the workflow followed throughout the project is illustrated in Figure 18. The approach to solving this problem can be divided into five main categories: configuration of the simulation, datasets generation, data preprocessing, network and model building, and model training and evaluation.

4.1 Simulation Configuration

This project focuses on classifying a rare phenomenon, the detection of WIMPs in liquid xenon detectors. As DM remains undetected, real experimental data is not available. Instead, simulated data is used, generated through a program that models interactions based on theoretical frameworks and detector physics.

The simulator ([Aristizabal Sierra et al., 2019]), provided by an expert in astroparticle and neutrino physics, generates events by modeling interactions of potential DM signals and neutrino backgrounds with the liquid xenon medium. This includes contributions from standard neutrino noise as well as hypothetical non-standard phenomena related to new vectorial interactions of neutrinos.

The output of the simulator is a histogram, or an array with the number of events according to the binning of the energy recoil based on the Er_{min} and Er_{max} parameters⁴¹. The output histogram (or array) can also be understood as an scenario. In Figure 19 the output of the simulator is depicted.

To use the simulated data effectively, the simulator's functionality was modified to automatically generate multiple scenarios instead of providing results for a single, predefined case, thus enabling the generation of files with several scenarios (rows). Each scenario varies based on two adjustable parameters, which are discussed in the following subsections.

⁴¹ Er_{min} and Er_{max} are fixed parameters explained on Subsection 4.1.1. With these parameters fixed, the binning done inside the simulator is also fixed for every scenario, and the comparison can be conducted only over the number of events.

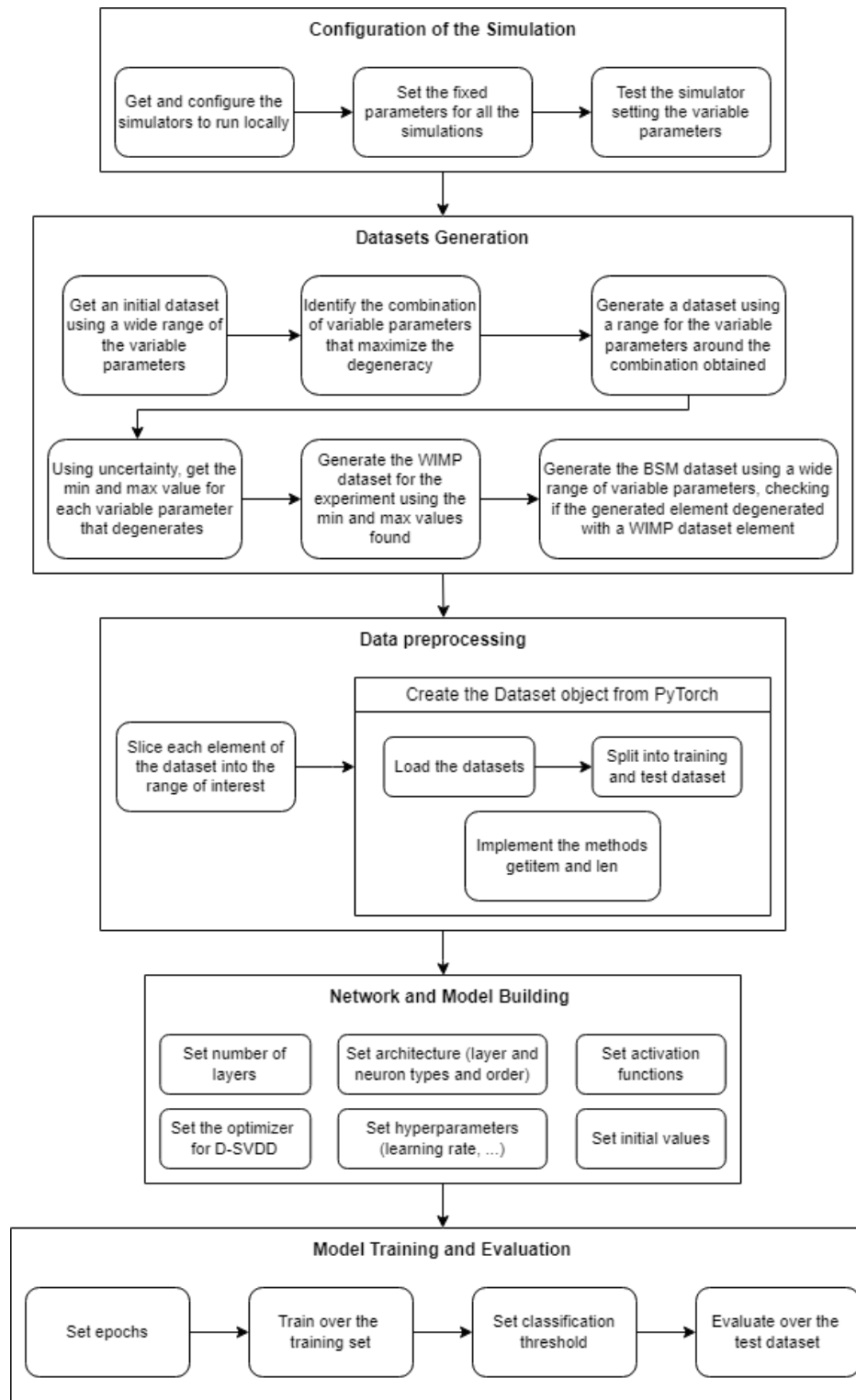


Figure 18: Workflow followed in the project and proposed solution. Image source: Author's creation.

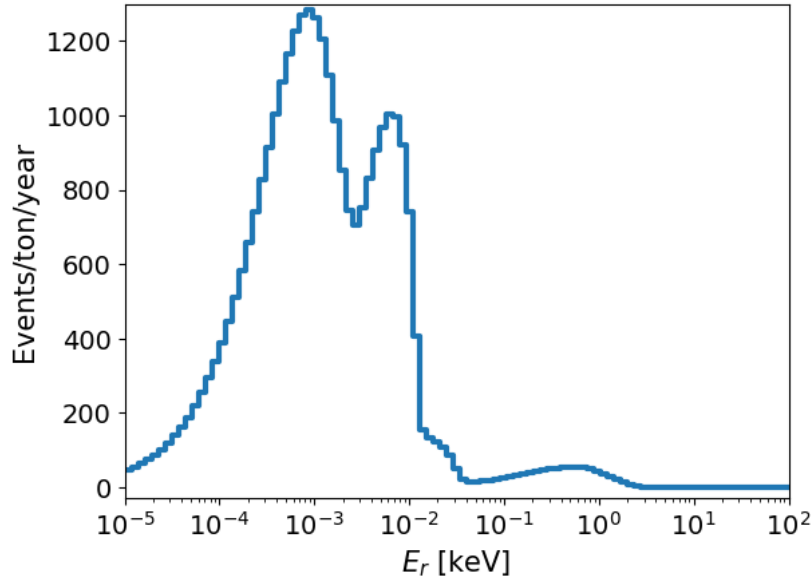


Figure 19: Example of the plot that can be done with the output of the simulation of the standard neutrinos. Image source: Author's creation.

4.1.1 Simulation Fixed Parameters

The simulation relies on a set of fixed parameters that remain constant across all scenarios. These parameters define the baseline conditions of the detector and the physical environment, ensuring consistency in the generated data. By holding these parameters fixed, the simulator provides a stable foundation for studying the effects of varying other parameters.

- **Er_min:** Minimum energy recoil to consider. Measured in keV.
- **Er_max:** Maximum energy recoil to consider. Measured in keV.
- **mdet:** Mass of the detector, amount of detection material. Measured in kg.
- **exptime:** Exposure time, the amount of time the detector is collecting data. Measured in years.
- **n_bins:** Number of bins to consider in the generation of the scenario histogram. Measured in units.
- **mN:** "Mass of the nucleus", the atomic weight of the atom used as detection material. Measured in GeV.
- **A:** Mass number of the atom used as detection material. Measured in u^{42} .
- **Z:** Atomic number of the atom used as detection material. Measured in u.

Table 1: Values set for each fixed parameter.

Parameter	Used Value
Er_min [keV]	$1 \cdot 10^{-5}$
Er_max [keV]	$1 \cdot 10^2$
mdet [kg]	$1 \cdot 10^3$
exptime [yr]	1.0
n_bins	100
mN [GeV]	122.3873664
A [u]	131.28734
Z [u]	54

The values for the fixed parameter used in the simulations are present in Table 1.

4.1.2 Simulation Variable Parameters

In addition to the fixed parameters, the simulation incorporates a set of variable parameters to explore different scenarios. These parameters are adjusted between simulations to account for the different values the properties of the particles could have according to the theoretical models.

- **mDM**: Dark matter mass, the possible value for the WIMP mass. Measured in GeV.
- σ_W : Differential cross section of the WIMP particle. Measured in cm^2 .
- **mV**: Mass of the hypothetical vectorial mediator for the new neutrino interaction. Measured in MeV.
- $C_V^N F_V$: The coupling strength of the new hypothetical neutrino interaction. Dimensionless.

The simulator uses the mDM and the σ_W parameters for the WIMP simulation, while it uses mV and $C_V^N F_V$ for the BSM ν simulation.

The intervals used in the first set of the simulations are present in Table 2, while the values for the second set of simulations are present in Table 3.

4.1.3 Definition of the Simulations

To develop and validate the model, five distinct simulations were performed: one for standard neutrinos, two for WIMPs, and two for BSM ν . Each simulation produced a defined

⁴²Unit called dalton or unified atomic mass unit with symbols Da or u, respectively

Table 2: Values set for each variable parameter in the first set of simulations.

Parameter	Min Value	Max Value
mDM [GeV]	$1 \cdot 10^{-5}$	$1 \cdot 10^5$
σ_W [cm ²]	$1 \cdot 10^{-60}$	$1 \cdot 10^{-40}$
mV [MeV]	$1 \cdot 10^{-2}$	$1 \cdot 10^5$
$C_V^N F_V$	$1 \cdot 10^{-8}$	$1 \cdot 10^{-2}$

Table 3: Values set for each variable parameter in the second set of simulations.

Parameter	Min Value	Max Value
mDM [GeV]	$8.5 \cdot 10^{-2}$	$1.5 \cdot 10^6$
σ_W [cm ²]	$8.5 \cdot 10^{-48}$	$1.5 \cdot 10^{-41}$
mV [MeV]	$7 \cdot 10^0$	$9 \cdot 10^4$
$C_V^N F_V$	$5 \cdot 10^{-9}$	$3 \cdot 10^{-2}$

number of rows and output files.

Base Simulation: Standard Model Neutrinos

The first simulation conducted focused on standard neutrinos, serving as a baseline for comparison and the identification of degeneracy. Since this scenario adheres strictly to the SM, the simulation generates a single row representing the spectrum of Standard Solar Neutrino (SSN) interactions.

First Simulation Set: Parameter Space Exploration

The first simulations for both WIMPs and BSM ν focused on exploring a wide range of the parameter space. This phase aimed to identify the largest intervals where the signals generated by WIMPs or BSM ν exhibit degeneracy with the SM neutrino simulation.

- **WIMP Simulation:** Using the parameters from Table 2, 10^6 arrays divided into 100 files of 10^4 rows were generated.
- **BSM ν Simulation:** Using the parameters from Table 2, 193600 arrays divided into 44 files of 4400 rows were generated.

Second Simulation Set: Intensification

The second simulations for WIMPs and $\text{BSM}\nu$ were designed to refine the results from the first phase. By tuning the variable parameters based on the insights gained, this iteration concentrated on narrowing down the parameter space and increasing the number of rows that exhibit degeneracy with the standard neutrino signals.

- **WIMP Simulation:** Using the parameters from Table 3, $2.25 \cdot 10^6$ arrays divided into 50 files of 45000 rows were generated.
- **$\text{BSM}\nu$ Simulation:** Using the parameters from Table 3, 10^6 arrays divided into 50 files of $2 \cdot 10^4$ rows were generated.

4.2 Dataset Generation

The dataset generation process involves analyzing the simulated data and determining if each row exhibits degeneracy with the standard neutrino background. By identifying these degenerate cases, the dataset provides a comprehensive framework for testing the algorithm's ability to distinguish between WIMP signals, $\text{BSM}\nu$ signals, and standard neutrino noise.

For each WIMP and $\text{BSM}\nu$ simulation defined on Subsection 4.1.3 several CSV⁴³ files were generated⁴⁴. The simulated data is organized into columns, where columns 0 through 99 contain the number of events per bin, representing the histogram of event distributions for each scenario. In the WIMP simulation, columns 100 and 101 are labeled **mDM** (DM mass) and **sW** (cross section), respectively, representing the values of the variable parameters used to generate the corresponding histogram. Similarly, in the $\text{BSM}\nu$ simulation, the last two columns, labeled **CVNFV** (coupling strength) and **mV** (mediator mass), indicate the values of the variable parameters specific to those scenarios.

Since the data is divided across different files, a new file is generated to get the information about the degeneracy (Subsection 4.2.1) and eligibility (Subsection 4.2.2) of each row to be used as the final dataset. The columns of this new file are the following:

- **file:** The name of the file where the analyzed row is stored.
- **index:** The index inside the file of the analyzed row.
- **Variable Parameter 1:** if WIMP then **mDM**, else **mV**.

⁴³Comma-separated values.

⁴⁴Specifically, $100 + 44 + 50 + 50 = 244$ files were generated.

- Variable Parameter 2: if WIMP then **sW**, else **CVNFV**.
- **degenerate**: Column indicating if the degeneracy condition is met.
- (Only for WIMP) **max**: Maximum amount of events across the bins.
- (Only for BSM ν) **1eventSum**: Difference between the sums over number of events of the histogram and the neutrino background.
- (Only for BSM ν) **1eventMax**: Maximum amount of events after applying the difference across the bins of the histogram with the neutrino background.

Taking into consideration the conditions of degeneracy and eligibility, four files were created⁴⁵:

- WIMP_DEGENERACY.csv
- WIMP_DEGENERACY_MK2.csv
- BSM_NU_DEGENERACY.csv
- BSM_NU_DEGENERACY_MK2.csv

4.2.1 Degeneracy Analysis

Each row in the simulated data is assessed for degeneracy, defined as the degree of similarity between the signal and the standard neutrino background. This analysis involves comparing the spectral features of the simulated signal (either WIMP or BSM ν) with those of the standard neutrinos. Rows exhibiting high similarity, where signals are nearly indistinguishable, are marked as degenerate.

Degeneracy between the simulated data and the standard background is evaluated by incorporating uncertainty into the comparison process. Specifically, the expected values of the simulated signals are compared against the SSN background, taking into account the uncertainty associated with the latter. Degeneracy is confirmed when the expected value of a simulated signal falls within the uncertainty interval of the SSN background.

The uncertainty used in this work is the square root of the number of events, as it's considered that the events behave following a Poisson distribution. The uncertainty interval is then $[n - \sqrt{n}, n + \sqrt{n}]$, where n is the event number of a bin. See the appendix for a mathematical formulation of the degeneracy function.

⁴⁵“MK2” refers to the second iteration or set of simulations.

Using the vocabulary of this work, the degeneracy takes two arrays and returns 1 if each element of the first array is within the uncertainty interval of the second array, and is equal to 0 otherwise. Thus, if $deg(\mathbf{x}, \mathbf{y})$ denotes the degeneracy function, and ssn represent the histogram of SSNs, the condition of degeneracy for a histogram $\mathbf{x}$ is:

$$deg(\mathbf{x}, ssn) = 1 . \quad (15)$$

4.2.2 Eligibility of the Data

The simulator returns a continuous value for the number of events, giving the expected event distribution across the histogram bins.

For the WIMP simulation, the generated data exclusively represents WIMP events, without contributions from other sources. To determine if a given scenario is detectable, a condition is applied: at least one bin in the histogram must have a value of one or more events. This threshold ensures that scenarios with negligible event rates, which are unlikely to be distinguishable from noise, are excluded from further analysis. Mathematically, this condition for the WIMP simulation can be expressed as:

$$\max(A) \geq 1 , \quad (16)$$

where $A = \{a_1, \dots, a_n\}$ is a set of real numbers that represent the WIMP histogram in this case.

In contrast, the $BSM\nu$ simulation incorporates both the SM neutrino interactions and the additional contributions from the hypothetical new interaction⁴⁶. To assess the detectability of a scenario in this case, an element-wise absolute comparison is performed between the simulated histogram and the SSN background. Detectability is confirmed if there is at least one bin with difference of one event. This approach accounts for the combined signal and ensures that the additional interaction yields a significant deviation from the background, highlighting scenarios where the presence of new physics might be observable. Given a $BSM\nu$ histogram $B = \{b_1, \dots, b_n\}$, and the SSN histogram $S = \{s_1, \dots, s_n\}$, the condition for the $BSM\nu$ simulation is expressed as:

$$\exists i \in \{1, \dots, n\} / |b_i - s_i| \geq 1 . \quad (17)$$

⁴⁶This is the way the simulator is designed.

4.3 Data Preprocessing

The process of data preprocessing involves filtering the raw data, narrowing it down to relevant rows, focusing on specific bins that contain detectable events, and implementing an interface for model training.

4.3.1 Filtering the Raw Data

The initial step in preprocessing involves filtering the raw simulated data to isolate rows that meet degeneracy and detectability conditions. As described previously, the simulations require that at least one bin presents one event associated to new physics (WIMP or ν new interaction). The files listed on Subsection 4.2 have the data for the filtering, so this filter step of preprocessing consist on joining the data from those files with the raw data from the second set of simulations defined on Subsection 4.1.3.

After applying the filtering conditions, the WIMP dataset was reduced to 179542 rows, representing 7.98% of the total rows generated in the second simulation. Similarly, the BSM ν dataset was narrowed down to 111319 rows, accounting for 11.13% of the rows from its second simulation.

4.3.2 Bin Selection Under ^8B

The detectable events in the dataset are concentrated within the energy range influenced by ^8B neutrinos, the second most energetic SSN source (recall Figure 13). Since the other solar processes produce low energy neutrinos, only the bins under the ^8B influence are kept. Specifically, this reduces the amount of bins from 100 to 22.

4.3.3 Dataset Interface Implementation

The last step in the preprocessing is the implementation of the Pytorch Dataset Interface for the WIMP dataset and the BSM ν dataset. This implementation enables streamlined data handling, allowing for batch processing and dynamic data loading during training. By encapsulating the preprocessed dataset within this interface, the model training pipeline becomes both modular and scalable.

Within the implementation, the following was made:

- Set the name of the file with the data.
- Code the data import.

- Split the data into training and testing data with a rate of 70% for the training data.
- Reshape the data so it can be converted into a tensor.
- Implement the `__len__` method which gives the amount of examples present.
- Implement the `__getitem__` method which gives the histogram array, the label of the array, and the index of the requested item.

For the subsequent evaluation of the model, the implementation of the WIMP dataset incorporates both SM neutrinos and shifted arrays derived from the SM neutrino uncertainty. Similarly, the implementation of the $\text{BSM}\nu$ dataset integrates data from both WIMPs and $\text{BSM}\nu$ scenarios, as both degenerate with the ν -floor. This approach aligns with the primary objective of this work: to identify potential signs of BSM physics within the uncertainty interval of the SM neutrinos, where degeneracy occurs.

4.4 Network and Model Building

The first dataset, composed of 99% WIMP events, exhibits a significant class imbalance, so pronounced that the problem can effectively be framed as a one-class classification task. Consequently, the identification of signals is approached as an AD problem, for which the Deep-SVDD method, introduced in 3.2.8, is implemented as the proposed solution.

The model is structured as an ANN that maps the WIMP class to a region near the center of a hypersphere, while mapping all other signals outside the hypersphere. During the training process, the NN learns to project inputs onto a scalar value representing their distance from the hypersphere's center. In a subsequent step, a radius must be defined by the researcher to separate normal examples (inside the hypersphere) from anomalies (outside), as illustrated in Figure 9.

For parameter configuration, the recommendations outlined in the original Deep-SVDD paper [Ruff et al., 2018] were carefully considered, ensuring the model adheres to established best practices for this method.

4.4.1 Network Architecture

The model is implemented as a 1D CNN, suitable for processing the histogram-like structure of the input data as mentioned in Subsection 3.2.6. Figure 20 illustrates the architecture of the proposed CNN “wimpsv1” showing the input layer, the convolutional and max pooling layers (2 of each), the dense layer and the output layer. The details of the architecture are:

1. **Input Vector:** (1, 22)

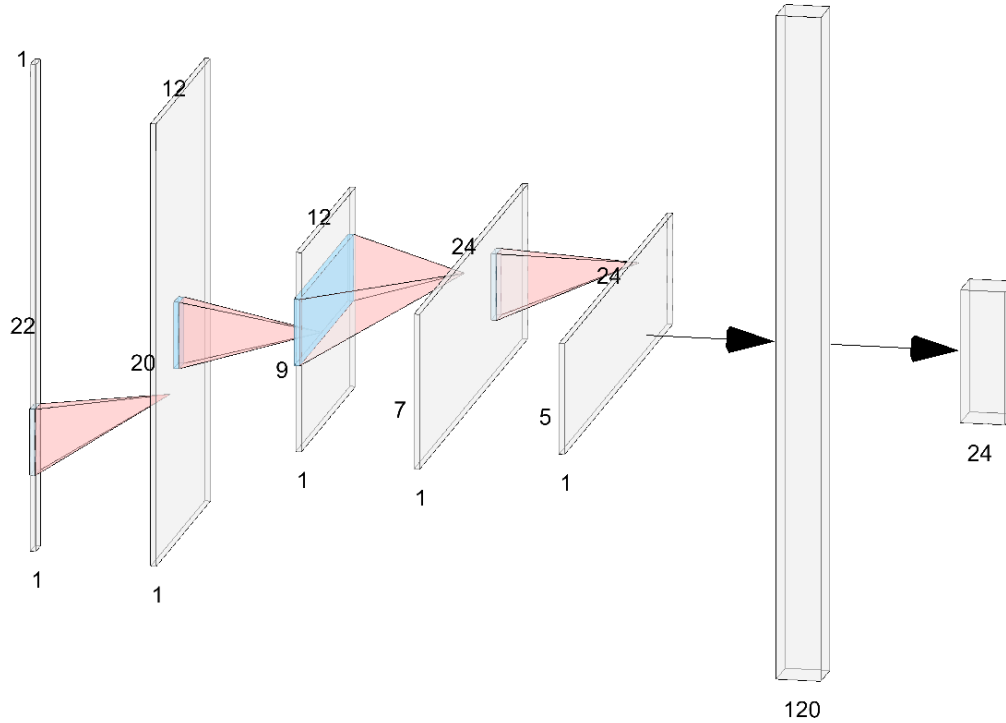


Figure 20: Architecture of the proposed Convolutional Neural Network “wimpsv1”, showing the amount of features and its length, as well as a depiction of the kernel of each layer. Image source: Author’s creation.

2. Convolution 1D: (12,20)

- bias = False (no bias used)
- Padding = 0
- Stride = 1
- kernel size = 3

Batch Normalization:

- $\epsilon = 10^{-4}$ (constant for numerical stability)
- affine = False (no training parameters used)

Leaky ReLU:

- slope = 0.01

3. Max Pool 1D: (12,9)

- Padding = 0
- Stride = 2

- kernel size = 3

4. **Convolution 1D:** (24,7)

- bias = False (no bias used)
- Padding = 0
- Stride = 1
- kernel size = 3

Batch Normalization:

- $\epsilon = 10^{-4}$ (constant for numerical stability)
- affine = False (no training parameters used)

Leaky ReLU:

- slope = 0.01

5. **Max Pool 1D:** (24,5)

- Padding = 0
- Stride = 1
- kernel size = 3

6. **Flatten:** (120,1)

7. **Linear (output) Vector:** (24,1)

- bias = False (no bias used)

In the listing above, only the layers of the architecture are enumerated, while other components that contribute to defining the architecture but do not correspond to layers are included without enumeration. The listing presents pairs of numbers, where the first number indicates the number of features or “channels”, and the second number represents the length of each feature.

4.4.2 Initialization and Hyperparameters

The network’s initial parameter values are set by the library used, ensuring a starting point that performs well for most cases.

1. **Optimizer:** ADAM (Subsection 3.2.5).

2. **Weight Initialization:** Default values assigned by Pytorch⁴⁷⁴⁸.
3. **Bias Initialization:** As suggested in [Ruff et al., 2018], no bias term was included to prevent “hypersphere collapse”.
4. **Initial Learning Rate:** 0.001 (or step size).
5. **Epochs:** 50 (number of times the training set will be completely traversed).
6. **Scheduler:** [30, 40, 45] (milestones (epoch number) at which learning rate is multiplied by $\gamma = 0.1$).
7. **Batch Size:** 128 (amount of examples to draw for mini-batch).
8. **Weight Decay:** 10^{-6} (hyperparameter for the penalization of the weights magnitude, helps improving generalization).

4.4.3 Model’s Threshold

After training the NN, each example from the dataset can be mapped to a scalar value representing the distance to the center of the model hypersphere. For the learned class, its examples will have a small distance, while any other class will have a greater distance.

A classification threshold is applied to the distance, allowing to effectively differentiate between the target class and anomalies, aligning with the goals of learn how to detect DM based on theoretical models of WIMPs.

⁴⁷<https://pytorch.org/docs/1.9.1/generated/torch.nn.Linear.html>

⁴⁸<https://pytorch.org/docs/1.9.1/generated/torch.nn.Conv1d.html>

Chapter 5

Solution Validation

5.1 Degeneracy Validation

To ensure the datasets align with the problem’s goals, a degeneracy validation process was performed. This involved verifying whether the generated data rows for WIMPs and BSM ν simulations overlapped with the uncertainty range of the standard neutrino background, in other words, that the constraint in Equation (15) is satisfied.

For WIMP data, the eligibility restriction from Equation (16) has to be met too. Thus, Figures 21 and 22 were generated after filtering the WIMP simulation defined in Tables 2 and 3 respectively.

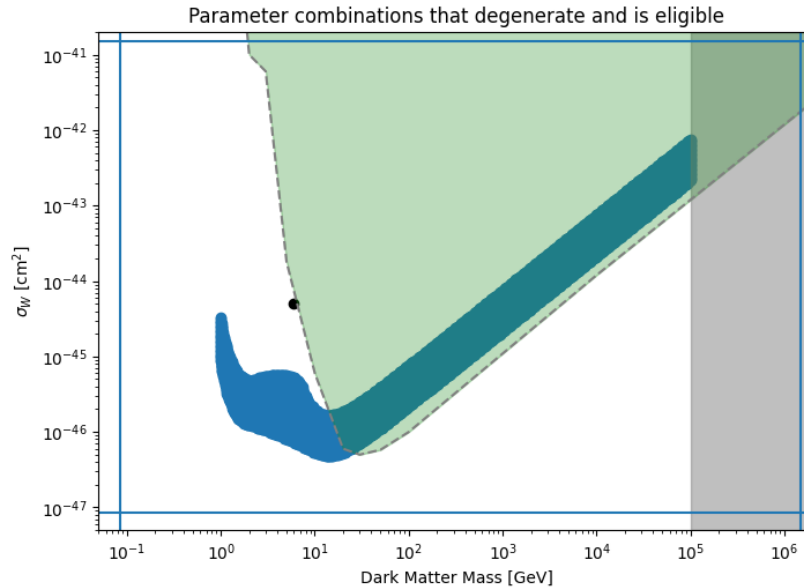


Figure 21: Parameter combination that produced degeneracy between the WIMP signals and the ν -floor from the first exploratory simulation. Green area indicates the explored limits as in [Billard et al., 2021], and the black dot indicates the point of maximum degeneracy as discussed in [O’Hare, 2016]. Image source: Author’s creation.

The only difference found between Figures 21 and 22 is the elongated “band” that spawns from 10^5 in Figure 22 and is missing in Figure 21.

The gray area depicted in Figures 21 and 22 shows the unitarity limit zone where the DM physics behave in a different manner and the process used in the detectors are no longer effective. This area is visited in Figure 22 because the conditions set in Equations (15) and (16) constraint the allowed combinations to follow a logarithmic law that can be extended

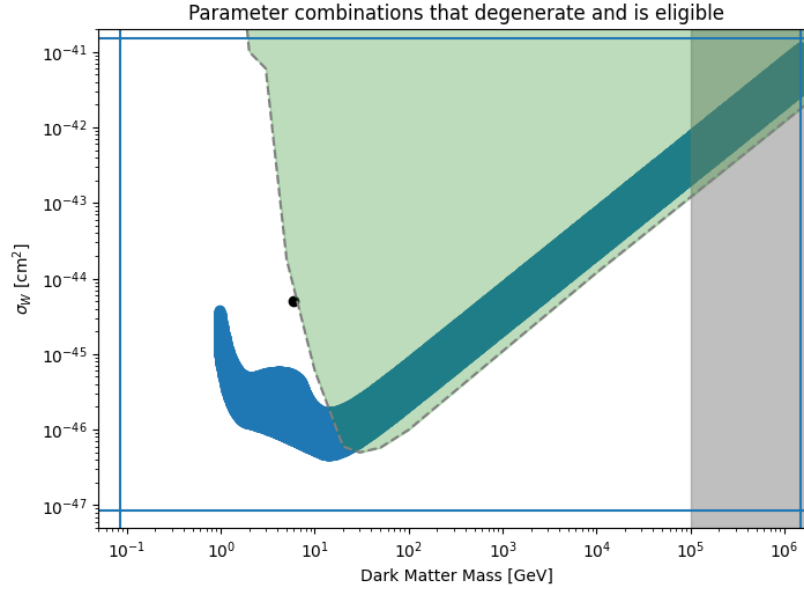


Figure 22: Parameter combination that produced degeneracy between the WIMP signals and the ν -floor from the second and final (intensification) simulation. Green area indicates the explored limits as in [Billard et al., 2021], and the black dot indicates the point of maximum degeneracy as discussed in [O’Hare, 2016]. Image source: Author’s creation.

using the simulator provided.

The green area in Figures 21 and 22 shows the parameter space region ruled out by experimental data as can be seen in 1 and discussed in [Billard et al., 2021]. In this case the area was visited by the algorithm due to the lack of the external input to avoid these combinations, meaning, the algorithm was not programmed to bypass these combinations.

The black dot in Figures 21 and 22 shows the parameter combination that presents complete degeneracy with the ν -floor under the considerations used in this work. This parameter combination is 6 GeV for WIMP mass, and $\sigma_W = 5 \cdot 10^{-45} [\text{cm}^2]$ (cross section), and is taken from [O’Hare, 2016]. The combination appears in the figures outside the region where the results of the simulation shows degeneracy because that combination does not satisfy the eligibility constraint from Equation (16). Although the combination satisfy the degeneracy constraint from Equation (15), as it does not produce a different expected value of event number, the combination was discarded by the algorithm implemented in this work.

The data used for the WIMP dataset and the one the NN was trained with, is the one generated with the parameter combination shown in Figure 22.

For $\text{BSM}\nu$ data, the eligibility restriction from Equation (17) has to be met too. Thus, Figures 23 and 24 were generated after filtering the $\text{BSM}\nu$ simulation defined in Tables 2 and 3 respectively.

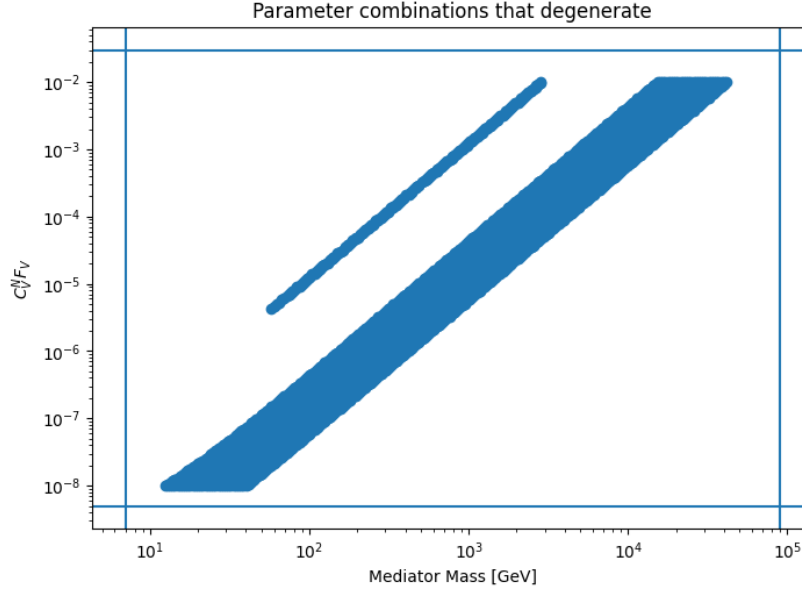


Figure 23: Parameter combination that produced $\text{BSM}\nu$ signals from the first exploratory simulation. Image source: Author's creation.

As in the WIMP case, the only difference found between Figures 23 and 24 is the extension of the spawned “band” in Figure 24. Similarly, the band of parameter combinations produced follow a logarithmic law.

The data used for the $\text{BSM}\nu$ dataset and the one the NN was trained with, is the one generated with the parameter combination shown in Figure 24.

The degeneracy validation ensured that the datasets consisted exclusively of scenarios relevant to the classification task.

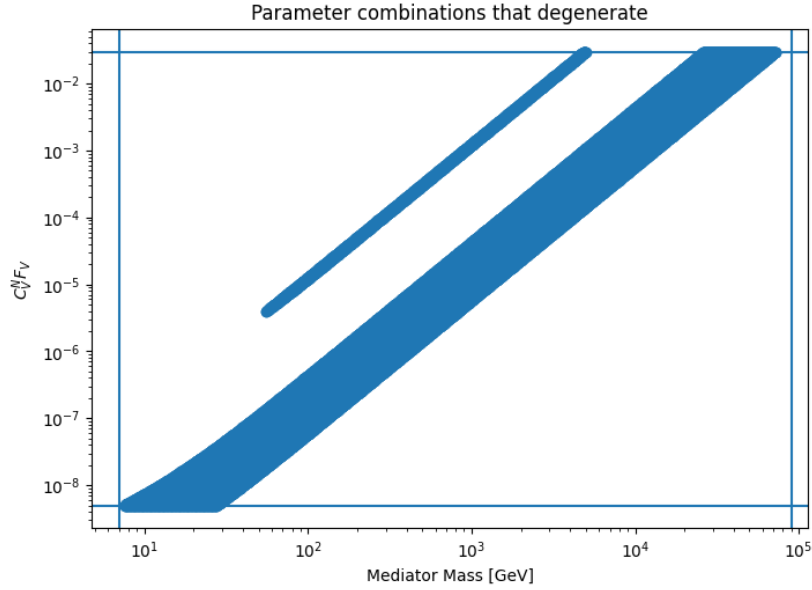


Figure 24: Parameter combination that produced $\text{BSM}\nu$ signals from the second and final intensification simulation. Image source: Author's creation.

5.2 Deep-SVDD Training

The NN was trained using the filtered datasets, with the architecture and configurations based on the Deep-SVDD method as discussed in Subsection 4.4. The training was carried out using the custom loss function from Equation (9), and minimizing the cost function in Equation (10).

The results of the training under the SM and BSM neutrino assumptions are shown in Figures 25 and 25 respectively.

To achieve a statistical significance, the NN was trained 11 times under each assumption (SM or $\text{BSM}\nu$), generating different parameter sets the NN can evaluate with. Thus, the error bar shown in the training figures, was made using the standard deviation of the 11 parameter set.

It is clear from the training figures that the reduction of the learning rate product of the scheduler (as defined in Subsection 4.4.2) induces an improvement in the loss value causing the plotted line to “step down”.



Figure 25: Training loss of the model trained with WIMP and Standard Model of Particle Physics Neutrino. Image source: Author's creation.

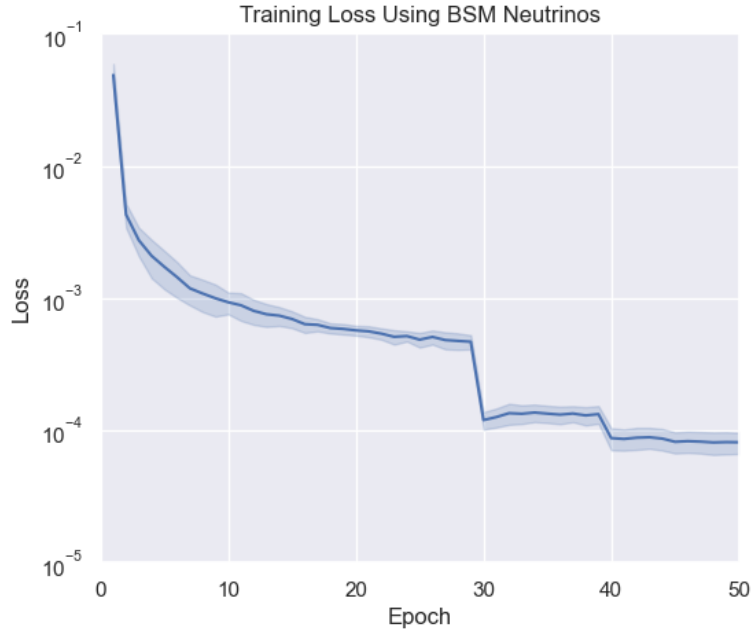


Figure 26: Training loss of the model trained with WIMP and Beyond Standard Model Neutrino. Image source: Author's creation.

5.3 Deep-SVDD Performance

The network's performance was assessed using standard and problem-specific metrics, namely, the AUC and the statistical metric defined in Equation (11).

The results of the training under the SM and BSM neutrino assumptions are shown in Figures 27 and 28 respectively, where a global optimal threshold of 10^{-2} was defined across every iteration to decide whether a datapoint belongs to the normal class or the anomaly class.

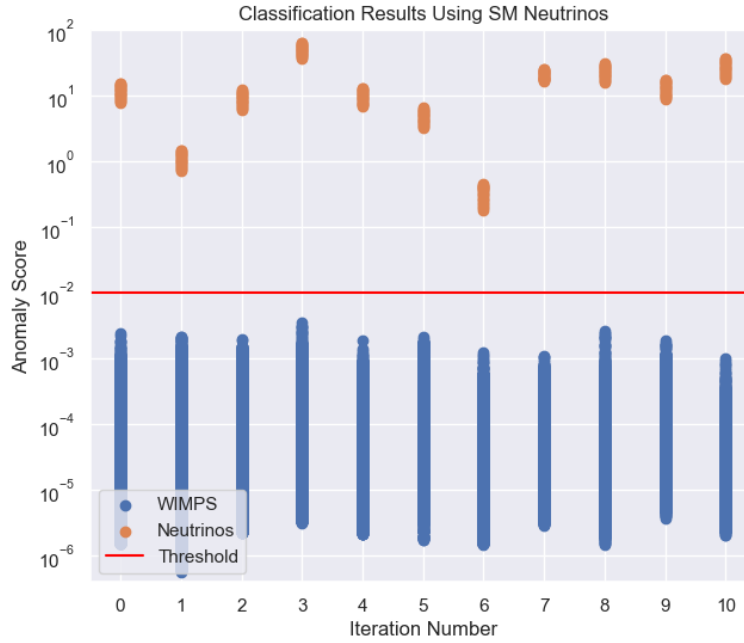


Figure 27: Classification results of the model trained with WIMP and Standard Model of Particle Physics Neutrino over 11 different iterations. Image source: Author's creation.

The threshold is determined by comparing the maximum anomaly score (or distance) value of the normal class with the minimum anomaly score value of the anomaly class. In essence, this approach evaluates the worst-case scenario based on the training data, ensuring the threshold effectively separates the two classes.

As can be seen in Figure 27, the 100% of the iterations trained with SM neutrinos could classify correctly the two classes, giving a mean AUC of 1, Equation (11), with standard deviation of 0. Although this is not the case for the classification with BSM ν as can be seen in Figure 28, resulting in a 91% of the iterations classifying successfully the two classes, with a mean AUC of 99.999%, Equation (11), and a standard deviation of 0.00003.

Although Figure 28 appears to show both classes overlapping, this is a result of the logarithmic scale used for the graph's axes. If we were to zoom in and examine the graph with a focus on the problem present in iteration 1, the separation between the classes would become evident. This distinction explains why the AUC for the test set is nearly 1. Consequently, instead

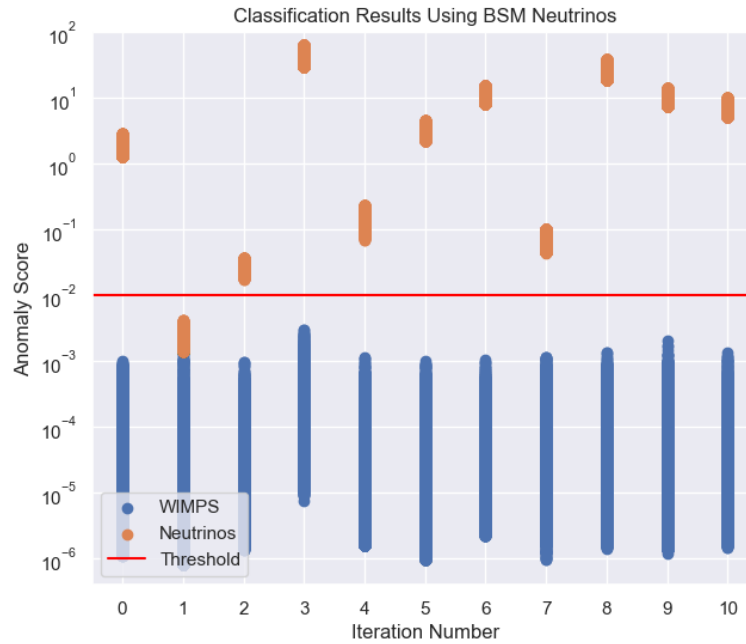


Figure 28: Classification results of the model trained with WIMP and Beyond Standard Model Neutrino over 11 different iterations. Image source: Author's creation.

of setting a global threshold over all iterations, focusing solely on this instance would allow to define a threshold that maximizes its classification performance.

Chapter 6

Conclusions

This work presented a DL based approach to address the challenge of identifying DM signals (WIMPs) and possible signs of BSM neutrino interactions in liquid xenon detectors. The implemented methodology focused on leveraging a one-class classification framework using the Deep-SVDD model to handle the significant class imbalance inherent in the dataset.

Here, a robust simulation pipeline was developed, integrating both fixed and variable parameters to generate realistic datasets for WIMPs and neutrinos, while a custom preprocessing pipeline validated and filtered the simulated data to include only degenerate scenarios relevant to the study. Additionally, a CNN adapted for one-class classification was designed, and statistical techniques such as AUC metrics were employed to rigorously evaluate the model performance.

These contributions demonstrate the feasibility of using DL to effectively distinguish DM signals, BSM neutrino interactions, and the SSM background, within uncertainty intervals that cause degeneracy. The approach achieved mean AUC scores of 1, with the exception of one out of 11 iterations of the $\text{BSM}\nu$ model, which yielded an AUC of 0.99.

Although the explored region of the parameter space includes areas where DM has already been studied and ruled out, it also uncovered an unexplored region where both degeneracy and detectability might occur. This is similarly true for the BSM vectorial neutrino interaction, where the results presented in this work identify the specific parameter space region where degeneracy and detectability with the SM are present.

One of the central findings of this work is the model's sensitivity to degeneracy in signals, particularly in distinguishing $\text{BSM}\nu$ and WIMP scenarios from the SSM background. The introduction of a threshold for classification based on the distance from a hypersphere center proved critical in balancing sensitivity and specificity.

However, the model's performance, while promising, also revealed challenges in handling scenarios with overlapping degeneracies, especially in cases where the uncertainty intervals between classes were minimal. Additionally, the reliance on simulated data, while unavoidable due to the lack of real DM detection events, introduces potential biases that could affect generalizability to real-world scenarios.

The evaluation metrics, particularly the AUC, highlighted the importance of carefully tuning hyperparameters and validating the model across multiple iterations to ensure statistical significance. Despite these limitations, the results provide a strong foundation for further exploration of DL methods in astroparticle physics.

6.1 Future Work

This work has effectively established a framework that demonstrates its utility in the context of identifying DM signals, serving as a proof of concept for the potential of DL methods in this field. In the early stages of designing new detectors, simulations play a critical role in evaluating their theoretical feasibility, providing a foundation for securing funding and support for the development of physical detectors.

While this study offers a phenomenological approach to the DM detection problem, it highlights the importance of integrating computational and theoretical methods as a precursor to experimental validation, bridging the gap between theoretical models and real-world implementations.

This study opens several promising avenues for further exploration and refinement. One key area for improvement lies in the simulation process. Incorporating more advanced simulation techniques or hybrid approaches that blend simulated data with real experimental data (as it becomes available) could significantly enhance the reliability and generalizability of the datasets used for training and evaluation. Expanding the model to alternative NN architectures or ensemble methods may further improve classification performance, especially in challenging cases where degeneracy is high, pushing the limits of the current framework.

Another important aspect is threshold optimization. Developing adaptive thresholding techniques capable of dynamically adjusting based on the data distribution in real-time could provide a more robust solution for classifying DM and neutrino signals. Additionally, collaborating with ongoing DM detection experiments would allow the validation of this model using experimental datasets, offering a critical step toward bridging the gap between simulation and reality. This integration could pave the way for practical applications of this framework in real-world detection experiments.

Finally, the methods and findings of this study could be generalized to other detector configurations and materials, broadening the framework's applicability across a diverse range of experimental setups. Such generalization could make the approach valuable in other domains of particle physics, extending its impact beyond liquid xenon detectors. By pursuing these directions, this work can evolve into a more comprehensive and versatile tool for advancing our understanding of DM and BSM physics, contributing meaningfully to the broader field of astroparticle physics.

APPENDIX

Activation Functions

Models of neurons are named according to its activation function:

- **Linear Neuron:** this kind of neuron tends to only be used in specific parts of the network. Its formulation is the following:

$$\sigma(x) = x$$

And graphically would look as the function in the Figure 29.

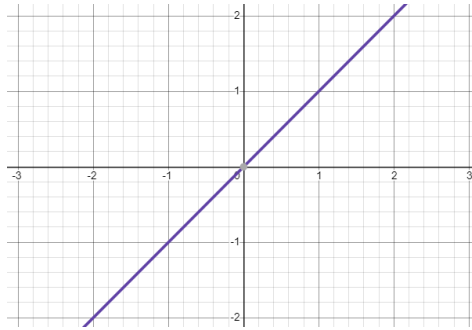


Figure 29: Linear activation function. Source: Author's creation.

- **Sigmoid Neuron:** this neuron can be considered a soft approximation (differentiable) of the classic binary activation function. This allows to train the neuron using gradient-based methods. Its formulation is the following:

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

And graphically would look as the function in the Figure 30.

- **Tanh Neuron:** it has been argued that allowing positive and negative activation values, this neuron offers more versatility. In comparison with the sigmoid neuron, negative values can be achieved with the activation function only and doesn't depend on the weights to achieve this. Its formulation is the following:

$$\sigma(x) = \frac{2}{1 + e^{-2x}} - 1$$

And graphically would look as the function in the Figure 31.

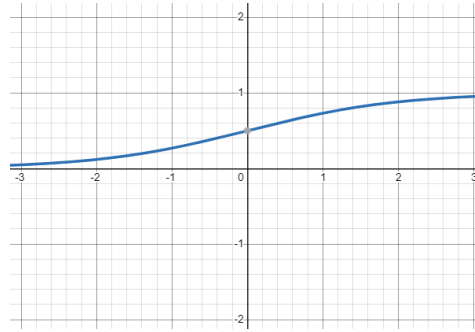


Figure 30: Sigmoid activation function. Source: Author's creation.

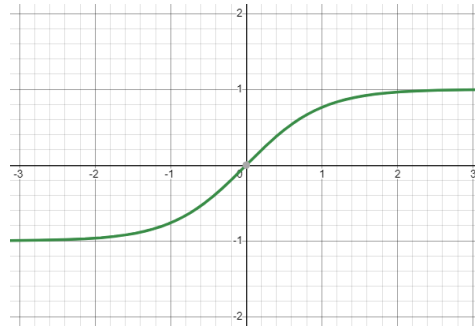


Figure 31: Tanh activation function. Source: Author's creation.

- **ReLU Neuron:** or rectifier linear units, is a more recent and popular option for the activation function [Glorot et al., 2011]. According to the authors, this function is a better approximation to the response of the biological neurons. This kind of neuron is very popular because it allows the introduction of sparse patterns of activation. The formulation of this neuron is the following:

$$\sigma(x) = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases} = \max(0, x)$$

And graphically would look as the function in the Figure 32.

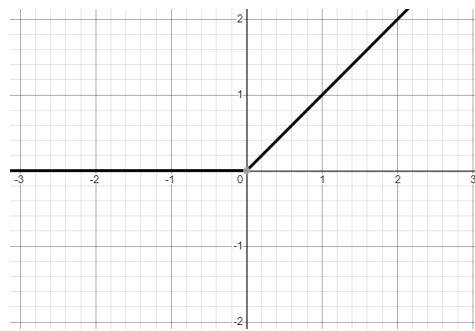


Figure 32: ReLU activation function. Source: Author's creation.

- **Leaky ReLU neuron:** the effectiveness of the ReLU neuron has led to a proliferation of variations on this model over the past decade. Most of these variations sacrifice the property of inducing sparse activations in the network to address issues that often arise during training, such as dead units. This is the case of the Leaky ReLU neuron, where instead of having a fixed zero for every negative value, it has a small window for negative values using the α parameter (or also called “leakiness”) that serves as a slope with a very small value, typically 0.01.

$$\sigma(x) = \begin{cases} x & x > 0 \\ x \cdot \alpha & x \leq 0 \end{cases} = \max(0, x)$$

And graphically would look as the function in the Figure 33.

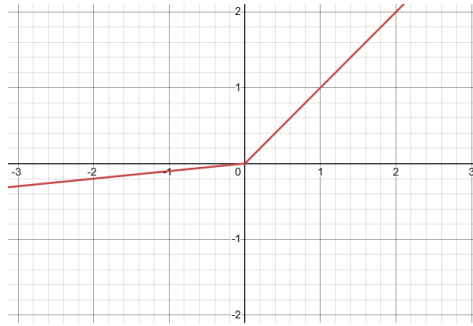


Figure 33: Leaky ReLU activation function, with $\alpha = 0.1$. Source: Author’s creation.

- **Softmax:** typically not plotted in the same way as functions like ReLU or sigmoid because it operates on a vector of inputs and produces a vector of outputs

$$\sigma(z_i) = \frac{e^{z_i}}{\sum_j e^{z_j}}$$

Degeneracy Analysis

The mathematical definition of the function that checks if two arrays degenerate is the following:

Definition 1 (Degeneracy). Let $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, with $\mathbf{x} = (x_1, x_2, \dots, x_n)$, $\mathbf{y} = (y_1, y_2, \dots, y_n)$. A function $deg : \mathbb{R}^n \times \mathbb{R}^n \mapsto 0, 1$ is defined by

$$deg(\mathbf{x}, \mathbf{y}) = \begin{cases} 1 & \text{if } y_i - \sqrt{y_i} \leq x_i \leq y_i + \sqrt{y_i}, \forall i \in \{1, 2, \dots, n\} \\ 0 & \text{otherwise} \end{cases}.$$

So, in the context of this work, $\mathbf{x}$ would be an array or histogram of the WIMP or BSM ν dataset, and $\mathbf{y}$ would be the histogram of the standard solar neutrinos.

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